4.6: Geometric Sums and Applications

Warm Up:

1. Evaluate $\displaystyle\sum_{n=1}^{478} 3n+2$

2. Write the series in sigma notation.

a) -7 + (-4) +(-1) + 2 + 5 + . . . + 98 + 101 + 104

b) 2 + 14 + 98 + 686 + . . . 1,647,086

We know how to find sums of finite arithmetic series. What about geometric series?

The Sum of a Finite Geometric Series:

The sum of the first n terms of a geometric series with common ratio r ≠ 1 is:

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

Example) Evaluate

$$\sum_{t=1}^{16} 4(3)^{t-1}$$

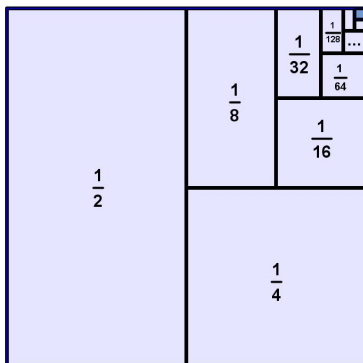Example) Given the sequence below, answer the following questions.

$$\begin{cases} p_1 = 7 \\ p_n = p_{n-1} \cdot 4, n > 1 \end{cases}$$

1. Find the explicit formula for the sequence.

2. Evaluate $\displaystyle\sum_{n=1}^{34} p_n$

Example) Consider a square with side lengths of 1.

The Sum of an Infinite Geometric Series:

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a}{1-r}$$

provided |r| < 1. If |r| ≥ 1, the series diverges (has no sum).

Example) Find the sum of the infinite geometric series.
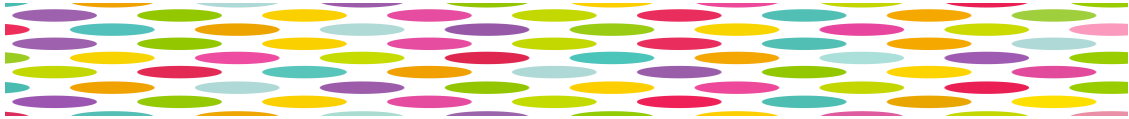
a) $\displaystyle\sum_{i=1}^{\infty} 5(0.8)^{i-1}$

b) $1 - \dfrac{3}{4} + \dfrac{9}{16} - \dfrac{27}{64} + \ldots$

Example) A pendulum that is released to swing freely travels 18 inches on the first swing. On each successive swing, the pendulum travels 80% of the distance of the previous swing. What is the total distance the pendulum swings?

Example) Elise is building a pyramid of paper cups. The bottom row has 72 cups while the top row has only one cup. If there are 18 total rows and the number of cups on each row represents an arithmetic sequence, find the total number of cups.

Example) The amount of interest that Michael earned on his savings account for the past three years since opening the account is shown in the table below. Determine whether this pattern represents an arithmetic or geometric sequence. Then, find the total amount of interest that Michael will earn after 25 years.

Year	Interest
1	$72
2	$86.4
3	$103.68

Classwork/Homework

PS 5.6