Chapter 7.2: Independent Events and Conditional Probability

Warm Up

Thirty AA batteries were tested to determine how long they would last. The results, to the nearest minute, were recorded as follows:

423, 369, 387, 411, 393, 394, 371, 377, 389, 409, 392, 408, 431, 401, 363, 391, 405, 382, 400, 381, 399, 415, 428, 422, 396, 372, 410, 419, 386, 390

Create a chart that gives the frequency and relative frequency of the battery life from the data set.

Battery life, (minutes)	Frequency	Relative Frequency	Percent Frequency
360 - 369			
370 - 379			
380 - 389			
390 - 399			
400 - 409			
410 - 419			
420 - 429			
430 - 439			

Example) A standard di is rolled two times. Find the probability that the sum of any two rolls will be **greater than or equal** to 7.

What would be the opposite of the event that the sum of the two rolls is greater than or equal to seven? What is its probability?

Notation

If we are talking about particular events E and F, then rather than write out the phrases:

"the relative frequency of E is 50%"

OR

" the probability of F is 5.7%"

WE WRITE

$$RF(E) = 0.5 \qquad P(F) = 0.057$$

Remember: relative frequencies and probabilities should be expressed as numbers between ________ and ___________ (inclusive).

Considering more than one event:

We rarely measure the Relative Frequency or Probability of an event in isolation. More often, we are concerned with the likelihood of a sequence of events, several events happening at the same time, or the effect of one event on another.

It is common in these situations to use a single letter to represent a particular event and symbol notation to represent the situation of interest.

When the outcome of one event DOES NOT AFFECT the outcome of another, we call them

INDEPENDENT EVENTS

Example)

Let S represent the event of a snow day

Let D represent the last digit of your phone number

Does event S affect D?

Does event D affect S?

Therefore, events S and D are called _________________.

When the outcome of one event DOES AFFECT the outcome of another, we call them

DEPENDENT EVENTS

Example)

Let H represent the number of hours you study for a test

Let G represent the grade you got on the test

Does H affect G?

Does G affect H?

Therefore, events S and D are called __________________.

The affect that dependent events have on each other may be large or small.

When two events are dependent, all we know is that they have SOME affect on each other, whether or not we can quantify it.

Can you come up with a pair of dependent events that have a LARGE affect on each other? What about a SMALL?

Classifying Events as Independent or Dependent

1. Selecting a king form a standard deck (A), not replacing it, and then selecting a queen from the deck (B).

2. Tossing a coin and getting a head (A), and then rolling a six-sided die and obtaining a 6 (B).

3. Driving over 85 miles per hour (A), and then getting in a car accident (B).

Conditional Relative Frequency or Probability

When considering events in a sequence, the outcome of one MAY be related to the outcome of another.

For example, considering lung cancer (L) and smoking (S). If you're a smoker, it significantly changes the probability that you will develop lung cancer.

We would call the probability that you develop lung cancer *GIVEN THAT* you are a smoker a **CONDITIONAL PROBABILITY**.

NOTATION:

P(L|S) ⟷ Probability of L given S

Here is the partial table of data from *The Three Stooges: An Illustrated History* by Michael Fleming (NY: Broadway Books, 1999):

Number of Slaps	Curly	Shemp	Joe	Total
0 to 5 slaps	20	9	5	34
6 to 10 slaps	29	25	5	59
11 to 15 slaps	19	16	2	37
16 to 20 slaps	16	5	2	23
21 to 25 slaps	3	6	1	10
26 to 30 slaps	4	7	0	11
31 to 35 slaps	1	2	0	3
36 to 40 slaps	2	0	0	2
More than 41 slaps	2	6	0	8
Total	96	76	15	187

Calculate

1. RF("Curly did the slapping" | "there were 26 to 30 slaps in the movie")

2. RF("Shemp did the slapping" | "there were 11 to 15 slaps in the movie")

Formal Definition of Independent Events

Events A and B are considered independent if and only if

$$P(A|B) = P(A)$$

and

$$P(B|A) = P(B)$$

This is a formal way of saying B doesn't affect A and A doesn't affect B, which is how we defined independence earlier.

Example) A six sided fair die is rolled and then a fair two sided coin is tossed. What is the probability that the result of the dice roll is even and a tail is flipped?

Example) A six sided fair die is rolled and then a fair two sided coin is tossed. What is the probability that the a tail is flipped **given that** the result of the dice roll was even?

Example) Two cards are selected in sequence form a standard deck. Find the probability that the second card is a queen, given that the first card is a king. (Assume the first card is **not** replaced).

Example) A study was conducted where researches examined the IQ's of 6 year olds along with the presence of lack of presence of a specific gene. 102 children were used in the study. The table shows the results of the study.

	Gene Present	Gene Not Present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

a) Find the relative frequency that a child has a high IQ, given that the child has the gene.

b) Find the relative frequency that a child does not have the gene.

c) Find the relative frequency that the child does not have the gene, given that the child has a normal IQ.

Classwork/Homework

Problem Set 7.2