7.3 Independent and Dependent Events, Conditional Probability

Independent Events	Definition:	Example:
Dependent Events	Definition:	Example:

Classifying events as independent or dependent:

1. Selecting a king form a standard deck (A), not replacing it, and then selecting a queen from the deck (B).

2. Tossing a coin and getting a head (A), and then rolling a six-sided die and obtaining a 6 (B).

3. Driving over 85 miles per hour (A), and then getting in a car accident.

Events A and B are considered independent **if and only if** _____ and _____.
(This is a formal way of saying B doesn't affect A and A doesn't affect B, which is how we defined independence earlier

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A **compound probability** is a probability involving _____ events, for example, the probability of Event A _____ Event B happening.

Probability Notation: There are multiple ways to ask the same question.

Words	Notation
A _____ B	P( )

TWO TYPES: _____replacement & _____ replacement

Example: Bag of Marbles: 5 red, 4 green, 8 blue, 3 pink striped   TOTAL:

1. If you select 1 marble, replace it, and select a 2nd marble, Find the probability that:

a) Both are Red   b) The first one is Red and the second one is Blue.

2. If you select 1 marble, keep it, and select a second marble, Find the probability that:

a) Both are Red   b) The first one is Red and the second one is Blue.

Example. What's the probability of flipping a coin twice and having it come up heads both times?

P(1st flip comes up heads AND 2nd flip comes up heads) =

Example. What is the probability of rolling a 4 on a die three times in a roll?

Example. What is the probability of drawing 3 hearts from a deck of cards without replacement?

Problem. There is a 20% chain of rain tomorrow. If it is raining, there is a 15% chance I will ride my bike. If it is not raining, there is a 70% chance I will go biking.

a) Complete a tree diagram.

b) What is the probability that it is raining and I did not ride my bike?

c) What is the probability that I will ride my bike?

Two events A and B (from the same sample space) are _____ when A and B have no outcomes in common.

Example. One card is selected at random from a standard deck of 52 cards. What is the probability that the card is either a heart or a face? What is the probability that the card is a heart and a face?
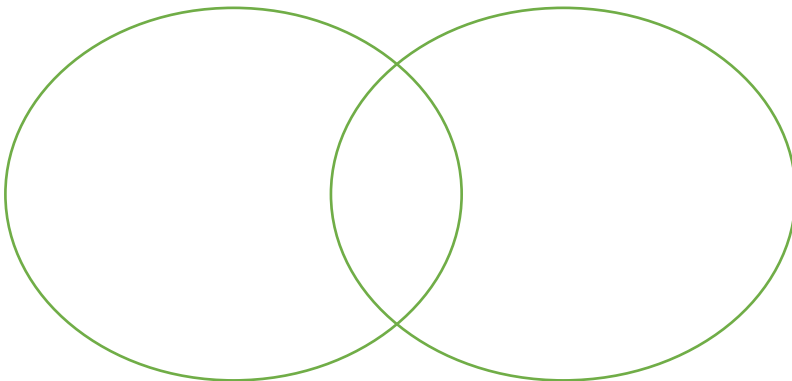
Probability of the Union of two Events
If A and B are events in the same sample space, then the probability of A or B occurring is given by

If A and B are mutually exclusive, then

Example. A card is drawn at random from an ordinary pack of 52 playing cards. Find the probability that the card is a heart or a king.

P(H) =   P(K) =   P(H ∩ K) =   P(H ∪ K) =

Example. Suppose $P(A) = 0.3, P(B) = 0.5$. Find $P(A \cup B)$ if…
a) the events are mutually exclusive   b) the events are independent

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Complementary Events
Events are _____ if their probabilities add up to _____ .
Examples: Tails is complementary to heads. A' *(not A)* is the complementary event of A.

Problems: The probability of rain on Tuesday of 0.2. What is the probability that it does not rain on Tuesday?

	Definition:	Example:
Conditional Probability		

The conditional probability of A given B is expressed as _____.

Example 1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

Formula:

Example 2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

Problem 1. In Ms. Schenkel's homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

Using Two-Way Frequency Tables to Compute Conditional Probabilities

1. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla.

Build a two-way frequency table that tracks the type of customers and type of ice cream.

	Vanilla	Chocolate	**Total**
Adult			
Child			
Total			

a) Find P(vanilla | adult) =   b) Find P(child | chocolate) =

2. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the table.

	Bus	Walk	Car	Other	**Total**
9th or 10th	106	30	70	4	
11th or 12th	41	58	184	7	
Total					

Suppose we randomly select one student.

a) P(student walked) =   b) P(9th or 10th grader) =   c) P(rode the bus OR 11th or 12th grader) =

d) What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

e) What is P(walk | 9th or 10th grade)?

Venn Diagram Example. A survey asked students which types of music they listen to. Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen to country music.