

Problem Set 4.1

1 - 3] For each of the following sequences, describe the sequence in words and then write standard recursive notation.

- 1) 100, 90, 80, 70, 60, 50, ... Each term is 10 less than the previous term

$$\begin{cases} a_1 = 100 \\ a_n = a_{n-1} - 10; n \geq 1 \end{cases}$$

- 2) 8, 16, 32, 64, 128, 256, ... Each term is double the previous term.

$$\begin{cases} a_1 = 8 \\ a_n = 2a_{n-1}; n \geq 1 \end{cases}$$

- 3) 10, 100, 10,000, 100,000,000, 10,000,000,000,000,000, ... ( $10^1, 10^2, 10^4, 10^8, 10^{16}, \dots$ ) Each term is

$$\begin{cases} a_1 = 10 \\ a_n = (a_{n-1})^2; n \geq 1 \end{cases}$$

$$10^1, (10^1)^2, (10^2)^2, (10^4)^2, (10^8)^2, \dots$$

being squaring the previous term.

- 4) Consider the sequence defined by  $\begin{cases} b_1 = 8,000 \\ b_n = b_{n-1} \cdot 1.007 - 329, n > 1 \end{cases}$

- a. Write the first 4 terms of the sequence (rounded to the nearest hundredth).

$$\begin{matrix} 8000, & 7727, & 7452.09, & 7175.25 \\ b_1 & b_2 & b_3 & b_4 \end{matrix}$$

- b. What is the 21<sup>st</sup> term of the sequence?

$$b_{21} = 2161.196$$

- 5) Consider the sequence defined by  $\begin{cases} b_1 = 180^\circ \\ b_n = \sin(b_{n-1}), n > 1 \end{cases}$

$$\sin(180^\circ) = 0 \quad \sin(0^\circ) = 0$$

- a. Write the first 4 terms of the sequence.

$$180^\circ, 0, 0, 0$$

- b. What is the 1,000<sup>th</sup> term of the sequence?

$$b_{1000} = 0$$

6) Jo Ann is learning Chinese. Currently she has memorized the meaning of 400 characters. Each month she forgets 10% of the characters she has learned (she remembers 90%) and learns 300 new characters.

- a. Write standard recursive notation for the sequence that represents the number of characters she knows each month.

$$\begin{cases} a_1 = 400 \\ a_n = 0.9a_{n-1} + 300; n \geq 1 \end{cases}$$

- b. If she continues to forget and learn as described above, what is the maximum number of characters she can learn? Explain why.

Since 10% of 3000 is 300, once she hits 3000 characters she will forget as many as she learns which will put her always knowing 3000 characters.