

Name: Key

Problem Set 4.2

1 - 3] Give the recursive and explicit notation for the arithmetic sequence.

1.) 7, 13, 19, 25, ...  
odd 6

Recursive:

$$\begin{cases} a_1 = 7 \\ a_n = a_{n-1} + 6; n \geq 1 \end{cases}$$

Explicit:

$$a_n = 6n + 1$$

2.) -3, -10, -17, -24, -31, ...  
subtract 7

Recursive:

$$\begin{cases} a_1 = -3 \\ a_n = a_{n-1} - 7; n \geq 1 \end{cases}$$

Explicit:

$$a_n = -7n + 4$$

3.)  $\frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \dots$   
add  $\frac{1}{3}$

Recursive:

$$\begin{cases} a_1 = \frac{2}{3} \\ a_n = a_{n-1} + \frac{1}{3}; n \geq 1 \end{cases}$$

Explicit:

$$a_n = \frac{1}{3}n + \frac{1}{3}$$

or

$$a_n = \frac{1}{3}(n+1)$$

4.) What is the 143<sup>th</sup> term of the sequence in number 2?

$$a_{143} = -7(143) + 4 = \boxed{-997}$$

5.) Find the **explicit formula** for the arithmetic sequence where  $a_{24} = 187$  and the common difference is 3.

$$\boxed{a_n = 3n + 115}$$

$$a_n = a_1 + 3(n-1)$$

$$187 = a_1 + 3(24-1)$$

$$187 = a_1 + 69$$

$$118 = a_1$$

$$a_n = 118 + 3(n-1)$$

$$a_n = 118 + 3n - 3 = \underline{3n + 115}$$

6.) Find the **explicit formula** for the arithmetic sequence where  $a_{10} = -26$  and  $a_{20} = -56$ .

$$\boxed{a_n = -3n + 4}$$

$$-26 = a_1 + d(10-1) \quad -56 = a_1 + d(20-1)$$

$$-26 = a_1 + 9d \quad -56 = a_1 + 19d$$

$$-(-56 = a_1 + 19d) \leftarrow$$

$$30 = -10d$$

$$-3 = d$$

$$-26 = a_1 + 9(-3)$$

$$-26 = a_1 - 27$$

$$+27 \quad +27$$

$$a_1 = 1$$

$$a_n = 1 - 3(n-1)$$

$$= 1 - 3n + 3$$

$$= \underline{-3n + 4}$$

7.) Find the **explicit formula** for the arithmetic sequence where  $b_4 = 19$ ,  $b_5 = 15$ , and  $b_6 = 11$ .

$$a_n = -4n + 35$$

$$d = -4$$

$$19 = b_1 - 4(4-1)$$

$$19 = b_1 - 12$$

$$\begin{array}{r} +12 \\ \hline 31 = b_1 \end{array}$$

$$b_n = 31 - 4(n-1)$$

$$= 31 - 4n + 4 = -4n + 35$$

8.) Find the **explicit formula** for the arithmetic sequence where  $a_7 = 4$  and  $a_{12} = -9$ .

$$a_n = -\frac{13}{5}n + \frac{111}{5}$$

$$-9 = a_1 + d(12-1) \quad 4 = a_1 + d(7-1)$$

$$-9 = a_1 + 11d \quad 4 = a_1 + 6d$$

$$-(4 = a_1 + 6d) \leftarrow$$

$$\begin{array}{r} -13 = 5d \\ \hline d = -13/5 \end{array}$$

$$4 = a_1 + 6(-13/5)$$

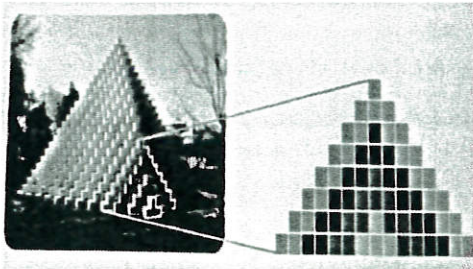
$$4 = a_1 - 78/5$$

$$\begin{array}{r} +78/5 \\ \hline 98/5 = a_1 \end{array}$$

$$a_n = \frac{98}{5} - \frac{13}{5}(n-1) = \frac{98}{5} - \frac{13}{5}n + \frac{13}{5}$$

$$= -\frac{13}{5}n + \frac{111}{5}$$

9.) Sol LeWitt's sculpture *Four-Sided Pyramid* in the National Gallery of Art Sculpture Garden is made of concrete blocks. As shown in the diagram below, each layer has 8 more visible blocks than the layer in front of it. Write an explicit formula that gives the number of visible blocks in the  $n^{\text{th}}$  layer where  $n = 1$  represents the front layer.



4, 12, 20, 28, ...

$$a_n = 8n - 4$$

$$d = 8 \quad a_1 = 4$$

$$a_n = 4 + 8(n-1)$$

$$= 4 + 8n - 8$$

$$= 8n - 4$$