Problem Set 4.3

1) This summer you have exactly 6 weeks free to work. Both your Mom and Dad have offered you a job. Your Mom offers you 1 penny today, 2 pennies tomorrow, 4 pennies the next day, growing geometrically for 6 weeks. Your Dad offers you $1,000 the first day, $1,100 the second day, $1,200, growing arithmetically for 6 weeks. Assume you work 6 weeks (5 days per week) for a total of 30 days. Calculate the total amount you would earn from your father.

\[
\text{Dad} \quad \quad 1000 + 1100 + 1200 + \ldots + 3900 = \sum_{n=1}^{30} 100n + 900 \\
\alpha_n = 1000 + 100(n-1) \\
= 1000 + 100n - 100 \\
\alpha_n = 100n + 900
\]

\[
\frac{30}{2} (1000 + 3900) = 73500
\]

2) If \( a_n = 2 + 3(n-1) \), find \( \sum_{i=1}^{10} 3i - 1 = \frac{90}{2} (2 + 26i) = 12195 \)

\[
\alpha_n = 2 + 3n - 3 \\
\alpha_n = 3n - 1
\]

3) If \( \left\{ \begin{array}{l} b_1 = -17 \\ b_n = b_{n-1} + 9, n > 1 \end{array} \right. \), find \( \sum_{i=1}^{1000} b_i = \sum_{i=1}^{1000} 9i - 2i = \frac{1000}{2} (-17 + 8974) \)

\[
= 500 (8957) = 4478500
\]
4) If \( c_n \) is an arithmetic sequence and \( c_2 = 106, c_3 = 89, c_4 = 72 \), find \( \sum_{i=1}^{25} c_i \) = \( \sum_{i=1}^{25} (-17i + 140) \)

\[
= \left( \frac{25}{2} \right) (132 + (-285)) \\
= 12.5 (-1102) \\
= -13775
\]

\[
\begin{align*}
c_1 &= 123 \\
c_n &= 123 - 17(n-1) \\
c_n &= 123 - 17n + 17 \\
c_n &= -17n + 140
\end{align*}
\]

5) A family has 5 children ages 4, 6, 8, 10, and 12. The parents have $100 that they are willing to give the children for allowance each month and are thinking of how they should divide up the $100. How much does each child get if the amount that they get is proportional to their age? (For example, the kid who is 12 will get twice as much as the kid who is 6.)

\[
\begin{align*}
4x + 1.5x + 2x + 2.5x + 3x &= 100 \\
10x &= 100 \\
x &= 10
\end{align*}
\]

\[
\begin{array}{c|c}
\text{Age} & \# \\
4 & 10 \\
6 & 15 \\
8 & 20 \\
10 & 25 \\
12 & 30
\end{array}
\]

6) In an arithmetic sequence \( r_3 = 207 \) and \( r_8 = 192 \).

a) Find the explicit formula for the arithmetic sequence.

\[
d = \frac{207 - 192}{3 - 8} = \frac{15}{-5} = -3
\]

\[
r_n = r_1 - 3(n-1)^2
\]

\[
\begin{align*}
r_n &= r_1 - 3 \cdot 3^2 \\
r_n &= r_1 - 27 \\
r_n &= r_1 + 6
\end{align*}
\]

b) Evaluate \( r_{13} \).

\[
r_{13} = -3(13) + 2110
\]

\[
= -39 + 2110
\]

\[
= 2071
\]

\[
r_1 = 213
\]

\[
r_n = -3n + 2110
\]

\[
S_{85} = \left( \frac{85}{2} \right) (213 + (-39)) = 7395
\]
7) Evaluate the series.

a) \( \sum_{k=1}^{42} k+152 = 151 + 150 + 149 + \ldots + 110 \)
\[ = \left( \frac{42}{2} \right)(151 + 110) \]
\[ = 21(261) \]
\[ = 5481 \]

b) \( \sum_{n=1}^{200} n = 1 + 2 + 3 + 4 + \ldots + 200 \)
\[ = \left( \frac{200 \cdot 201}{2} \right) \]
\[ = 100(201) \]
\[ = 20100 \]

c) \( \sum_{n=1}^{65} 2n+7 = 9 + 11 + 13 + \ldots + 137 \)
\[ = \left( \frac{65}{2} \right)(9 + 137) \]
\[ = 4745 \]

d) \( \sum_{n=1}^{4} 3n^2 - 5 \)
\[ \text{NOT ARITHMETIC} \]
\[ = 70 \]

e) \( \sum_{n=1}^{35} 3n+7 = 10 + 13 + \ldots + 82 \)
\[ = \left( \frac{35}{2} \right)(10 + 82) \]
\[ = 1150 \]

f) \( \sum_{n=5}^{25} n+7 = 22 + 25 + 28 + \ldots + 82 \)
\[ = \left( \frac{21}{2} \right)(22 + 82) \]
\[ = 1092 \]