

Name: Key

Problem Set 4.3

- 1) This summer you have exactly 6 weeks free to work. Both your Mom and Dad have offered you a job. Your Mom offers you 1 penny today, 2 pennies tomorrow, 4 pennies the next day, growing **geometrically** for 6 weeks. Your Dad offers you \$1,000 the first day, \$1,100 the second day, \$1,200, growing **arithmetically** for 6 weeks. Assume you work 6 weeks (5 days per week) for a total of 30 days. Calculate the total amount you would earn from your **father**.

Dad

$$1000 + 1100 + 1200 + \dots + 3900 = \sum_{n=1}^{30} 100n + 900$$

$$a_n = 1000 + 100(n-1)$$

$$= 1000 + 100n - 100$$

$$a_n = 100n + 900$$

$$= \left(\frac{30}{2}\right)(1000 + 3900)$$

$$= \boxed{\$73500}$$

2) If $a_n = 2 + 3(n-1)$, find $\sum_{i=1}^{90} a_i = \sum_{i=1}^{90} 3i - 1 = \left(\frac{90}{2}\right)(2 + 269) = \boxed{12,195}$

↓

$$a_n = 2 + 3n - 3$$

$$a_n = 3n - 1$$

3) If $\begin{cases} b_1 = -17 \\ b_n = b_{n-1} + 9, n > 1 \end{cases}$, find $\sum_{i=1}^{1000} b_i = \sum_{i=1}^{1000} 9i - 26 = \left(\frac{1000}{2}\right)(-17 + 8974)$

↓

$$b_n = -17 + 9(n-1)$$

$$b_n = -17 + 9n - 9$$

$$b_n = 9n - 26$$

$$= 500(8957)$$

$$= \boxed{4,478,500}$$

4) If c_n is an arithmetic sequence and $c_2 = 106$, $c_3 = 89$, $c_4 = 72$, find $\sum_{i=1}^{25} c_i = \sum_{i=1}^{25} -17i + 140$

123
 \swarrow \searrow \swarrow \searrow
 106, 89, 72, ...
 +17 -17 -17

$$= \left(\frac{25}{2}\right)(123 + -205)$$

$$= 12.5(-162)$$

$$= \boxed{-2025}$$

$c_1 = 123$

$c_n = 123 - 17(n-1)$

$c_n = 123 - 17n + 17$

$c_n = -17n + 140$

5) A family has 5 children ages 4, 6, 8, 10, and 12. The parents have \$100 that they are willing to give the children for allowance each month and are thinking of how they should divide up the \$100. How much does each child get if the amount that they get is **proportional to their age**? (For example, the kid who is 12 will get twice as much as the kid who is 6.)

(4) (6) (8) (10) (12)
 $x + 1.5x + 2x + 2.5x + 3x = 100$

$10x = 100$

$x = 10$

age	\$
4	10
6	15
8	20
10	25
12	30

6) In an arithmetic sequence $r_3 = 207$ and $r_8 = 192$.

a) Find the explicit formula for the arithmetic sequence.

$d = \frac{207 - 192}{3 - 8} = \frac{15}{-5} = -3$

$207 = r_1 - 3(3-1)$
 $207 = r_1 - 6$
 $+6 \quad +6$

$r_n = -3n + 216$

b) Evaluate r_{73} .

$213 = r_1$

$r_{73} = -3(73) + 216$
 $= -219 + 216 = \boxed{-3}$

c) Evaluate S_{85} for the arithmetic series.

$S_{85} = \left(\frac{85}{2}\right)(213 + -39) = \boxed{7395}$

\uparrow \uparrow
 r_1 r_{85}

7) Evaluate the series.

$$\begin{aligned} \text{a) } \sum_{i=1}^{42} -i+152 &= 151+150+149+\dots+110 \\ &= \left(\frac{42}{2}\right)(151+110) \\ &= 21(261) \\ &= \boxed{5481} \end{aligned}$$

$$\begin{aligned} \text{b) } \sum_{n=1}^{200} n &= 1+2+3+4+\dots+200 \\ &= \left(\frac{200}{2}\right)(1+200) \\ &= 100(201) \\ &= \boxed{20100} \end{aligned}$$

$$\begin{aligned} \text{c) } \sum_{n=1}^{65} 2n+7 &= 9+11+13+\dots+137 \\ &= \left(\frac{65}{2}\right)(9+137) \\ &= \boxed{4745} \end{aligned}$$

$$\begin{aligned} \text{d) } \sum_{n=1}^4 3n^2-5 &= -2+7+22+43 \\ &\quad \uparrow \\ &\quad \text{NOT} \\ &\quad \text{ARITHMETIC} \\ &= \boxed{70} \end{aligned}$$

$$\begin{aligned} \text{e) } \sum_{n=1}^{25} 3n+7 &= 10+13+\dots+82 \\ &= \left(\frac{25}{2}\right)(10+82) \\ &= \boxed{1150} \end{aligned}$$

$$\begin{aligned} \text{f) } \sum_{n=5}^{25} 3n+7 &= 22+25+28+\dots+82 \\ &= \left(\frac{21}{2}\right)(22+82) \\ &= \boxed{1092} \end{aligned}$$

