Name: Key

Problem Set 5.1

1. Instead of making a down payment on a house, a couple that lives in an apartment decides to invest \$50,000 that they inherited from Aunt Zelda into a real estate fund that earns 6.3% interest, compounded annually. Let A be the value of the fund after t years.

a) Write A as a function of t.

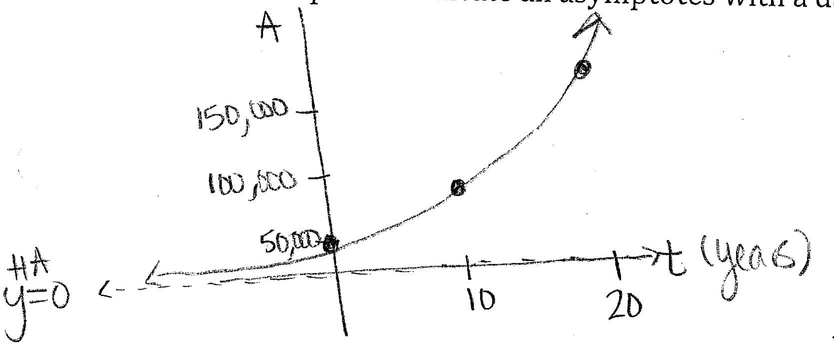
$$\boxed{A = 50{,}000(1.063)^t}$$

b) What will the value of the investment be after 10 years? 20 years?

10 years: $A = 50{,}000(1.063)^{10} = \boxed{\$92{,}109.12}$

20 years: $A = 50{,}000(1.063)^{20} = \boxed{\$169{,}681.81}$

c) Graph the function you wrote in (a) for years 0 through 20. Label the coordinates of the y-intercept and indicate all asymptotes with a dashed line.



d) Redo part (b), but use quarterly compounding instead of annual.

10 years: $A = 50{,}000\left(1 + \frac{.063}{4}\right)^{40} = \$93{,}420.73$

20 years: $A = 50{,}000\left(1 + \frac{.063}{4}\right)^{80} = \$174{,}548.65$

2. A couple has a baby and they want to put money in a college savings plan that assures them 7% interest for the next 18 years. If the parents want to have \$125,000 when their child starts college, how much do they need to put in this college savings program now? Note: For this problem and the rest of the course, if no mention is made of the type of growth (annually, compounded quarterly, continuously, etc) assume annually compounded growth.

$125{,}000 = a(1+.07)^{18}$

$\frac{125{,}000}{3.3799} = \frac{a \cdot \cancel{3.3799}}{\cancel{3.3799}}$

$\boxed{a \approx \$36{,}982.99}$

3. Redo (2), but assume the interest is compounded continuously.

$125{,}000 = Pe^{0.07(18)}$

$\frac{125{,}000}{3.5254} = \frac{P(\cancel{3.5254})}{\cancel{3.5254}}$

$\boxed{P = \$35{,}456.75}$

4. According to the CIA World Factbook, the population of Western Sahara is growing at a rate of 2.7%. Its current population is 605,253 people.

a) What will the population of Western Sahara be in one year?

$P = 605253(1.027)^1 = \boxed{621594.83 \text{ people}}$

b) What was the population one year before the most current estimate?

$P = 605253(1.027)^{-1} = \boxed{589340.79}$

c) Express the population P as a function of n, the number of years from now.

$\boxed{P = 605,253(1.027)^n}$

5. Make up a context or situation for which the relationship between x and y is y = 300*(1.02)ˣ.

answers will vary → starting amt must be 300 & grow at an annual rate of 2%.

6. For each description of an exponential function f(x) = a(b)ˣ, find a and b.

a) f(0) = 3 and f(1) = 12

$3 = a(b)^0$ $\quad 12 = a(b)^1$

$3 = a(1)$ $\quad \frac{12}{3} = \frac{3(b)}{3}$

$3 = a$ $\quad 4 = b$

$\boxed{f(x) = 3(4)^x}$

b) f(0) = 4 and f(2) = 1

$4 = a(b)^0$

$4 = a(1)$

$4 = a$

$1 = 4(b)^2$

$\sqrt{\frac{1}{4}} = \sqrt{b^2}$

$\pm\frac{1}{2} = b$

$\boxed{f(x) = 4\left(\frac{1}{2}\right)^x}$

b must be $b > 1$ so $b = \frac{1}{2}$

7. Fungus is growing exponentially in a Petri dish in a circular pattern according to the function f(x) = a(b)ˣ, where a is the initial area, b is the growth factor and f(x) is the area after x hours. Two hours after the start of the experiment the area of the fungus was 5 mm². After 4 hours the area was 17 mm².

(2, 5) $\quad$ (4, 17)

a) Give a function for the area in terms of time since the start of the experiment.

$\frac{17 = a(b)^4}{5 = a(b)^2}$ $\quad \sqrt{\frac{17}{5}} = \sqrt{b^2}$ $\quad 1.843 \approx b$ $\quad 5 = a(1.843)^2$ $\quad 1.47 \approx a$

$\boxed{f(x) = 1.47(1.843)^x}$

b) What was the area 7 hours after the start of the experiment?

$f(7) \approx 1.47(1.843)^7 \approx \boxed{106.17 \text{ mm}^2}$