

Name: Key

Problem Set 5.2

1. In order for consumers to be able to compare interest rates from one institution to the next, the government often requires that the Effective Annual Yield (EAY) be reported. The EAY is the annualized interest rate which comes from the $(1 + \frac{r}{n})^{nt}$ portion of the $A = P(1 + \frac{r}{n})^{nt}$.

EAY for interest compounded n times per year is $(1 + \frac{r}{n})^n - 1$.

EAY for interest compounded continuously is $e^r - 1$.

Which interest rate has the highest EAY? The lowest EAY?

a) 6.5% compounded annually

$$= (1 + 0.065) - 1$$

$$= \boxed{0.065} \text{ Lowest}$$

b) 6.4% compounded monthly

$$= (1 + \frac{0.064}{12})^{12} - 1$$

$$= \boxed{0.065911} \text{ Highest}$$

c) 6.3% compounded continuously

$$= e^{0.063} - 1$$

$$= \boxed{0.065027}$$

2. As a benchmark, people are often interested in how long it will take to double their money in a particular investment. Do the necessary calculations to fill in the table below. Pretend your original investment is \$100 if you need help getting started. Show all of your calculations in the space below.

Interest Rate Compounded Annually	Years to double your Money (round to the nearest year)	Product of Rate and Years it takes to Double
3.5%	$200 = 100(1.035)^x$ $2 = (1.035)^x$ $\sqrt{x = 20.15}$	$(3.5)(20.15) = \underline{70.5}$
7.0%	$200 = 100(1.07)^x$ $2 = (1.07)^x$ $\sqrt{x = 10.24}$	$(7)(10.24) = \underline{71.68}$
10.0%	$200 = 100(1.10)^x$ $2 = (1.1)^x$ $\sqrt{x = 7.27}$	$(10)(7.27) = \underline{72.7}$

3. Can you state a rule of thumb for doubling your money based on the work you did in #2?

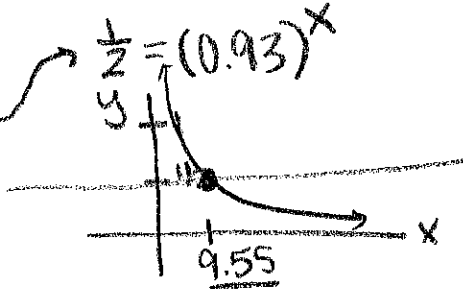
rate * years = about 70

4. According to the PEW Research Center, the portion of the US population that identifies themselves as Christian declined about 7% from 2007 to 2014. At this rate, how many years will it take until the number of those who identify as Christians is half of what it was in 2007? *Hint - you don't need to know the exact population in 2007 to answer this question*

Source: "America's Changing Religious Landscape." Religion and Public Life. PEW Research Center, n.d. Web 16 June 2015

Pretend started w/ 100.

$$50 = 100(0.93)^x$$



about 9.55 years

5. The Smithsonian Book of North American Mammals reports that, "Steller sea lion population numbers have declined by more than 90 percent in the last 20 years in most of Alaska and Southern California." If the population declined by the same percent each of those 20 years, by what percent did it decline each year? (Assume that the population declined by exactly 90%.)

Source: "The Smithsonian Book of North American Mammals (Natural History). P.199, Vancouver: Univ of British Columbia Pr, 2003.

Pretend started w/ 100

$$\frac{1}{10} = (b)^{20}$$

$$10 = 100(b)^{20}$$

But 10 is 10% of 100

$$b \approx 0.89 \text{ so } r = 0.11 \text{ or } \boxed{11\%}$$

6. If you buy a car for \$29,873 and after one year with typical driving distances it is only worth about \$27,314, what will the car be worth after 5 years (from its original purchase) if it continues to depreciate at the same rate?

$$y = 29873(b)^x$$

$$27314 = \frac{29873(b)^1}{29873}$$

$$0.914 = b$$

$$y = 29873(0.914)^x$$

$$y = 29873(0.914)^5$$

$$\approx \boxed{\$18641.11}$$

7. Find the exponential function $f(x) = a(b)^x$ such that $f(3) = 2$ and $f(5) = 32$.

$$2 = a(b)^3 \text{ and } 32 = a(b)^5$$

$$\frac{2}{2} = \frac{a(b)^5}{a(b)^3}$$

$$16 = b^2$$

$$4 = b$$

$$2 = a(4)^3$$

$$2 = a(64)$$

$$\frac{1}{32} = a$$

$$f(x) = \frac{1}{32}(4)^x$$

8. How could I evaluate the following expressions without a calculator? (Review)

a) $100^{\frac{1}{2}} = \sqrt{100} = \boxed{10}$

b) $100^{-\frac{1}{2}} = \frac{1}{100^{\frac{1}{2}}} = \frac{1}{\sqrt{100}} = \frac{1}{10} = \boxed{\frac{1}{10}}$

c) $16^{\frac{3}{2}} = \sqrt{16}^3 = 4^3 = \boxed{64}$

d) $27^{-\frac{4}{3}} = \frac{1}{27^{\frac{4}{3}}} = \frac{1}{\sqrt[3]{27^4}} = \frac{1}{3^4} = \boxed{\frac{1}{81}}$