

Problem Set 5.6

Solve the equations. Show all of your work in order to receive full credit.

$$1.) 5^{x-4} = 25^{x-6}$$

$$5^{x-4} = 5^{2(x-6)}$$

$$x-4 = 2(x-6)$$

$$x-4 = 2x-12$$

$$\boxed{8 = x}$$

$$2.) 36^{5x+2} = \left(\frac{1}{6}\right)^{11-x}$$

$$6^{2(5x+2)} = 6^{-1(11-x)}$$

$$10x+4 = -11+x$$

$$\frac{9x}{9} = \frac{-15}{9}$$

$$\boxed{x = -\frac{5}{3}}$$

$$3.) \log_4(x+3) - \log_4(x-5) = 2$$

$$\log_4 \frac{x+3}{x-5} = 2$$

$$4^2 = \frac{x+3}{x-5}$$

$$16(x-5) = x+3$$

$$16x - 80 = x+3$$

$$15x = 83$$

$$\boxed{x = \frac{83}{15}}$$

$$4.) 5^{2x+1} = 3^{x-5}$$

$$\ln 5^{2x+1} = \ln 3^{x-5}$$

$$(2x+1)\ln 5 = (x-5)\ln 3$$

$$2x\ln 5 + \ln 5 = x\ln 3 - 5\ln 3$$

$$2x\ln 5 - x\ln 3 = -\ln 5 - 5\ln 3$$

$$x(2\ln 5 - \ln 3) = -\ln 5 - 5\ln 3$$

$$\boxed{x = \frac{-\ln 5 - 5\ln 3}{2\ln 5 - \ln 3}}$$

$$5.) \log_4(-x) + \log_4(x+10) = 2$$

$$\log_4 -x(x+10) = 2$$

$$4^2 = -x^2 - 10x$$

$$x^2 + 10x + 16 = 0$$

$$(x+8)(x+2) = 0$$

$$\boxed{x = -8} \quad \boxed{x = -2}$$

$$6.) \log_5(5x+9) = \log_5(6x)$$

$$5x+9 = 6x$$

$$\boxed{9 = x}$$

$$7.) \frac{1}{3} \log_5(12x) = 2$$

$$\log_5 12x = 6$$

$$5^6 = 12x$$

$$\frac{5^6}{12} = x$$

$$x = 1302.083$$

$$8.) \log_2(x-4) = 5$$

$$2^5 = x-4$$

$$32 = x-4$$

$$\boxed{x = 36}$$

$$9.) 10^{x+4} = 3^{5x-1}$$

$$\log 10^{x+4} = \log 3^{5x-1}$$

$$(x+4)\log 10 = (5x-1)\log 3$$

$$x+4 = 5x\log 3 - \log 3$$

$$x - 5x\log 3 = -4 - \log 3$$

$$x(1 - 5\log 3) = -4 - \log 3$$

$$\boxed{x = \frac{-4 - \log 3}{1 - 5\log 3}}$$

$$10.) \log_2(x) + \log_2(x-2) = 3$$

$$\log_2 x(x-2) = 3$$

$$2^3 = x(x-2)$$

$$8 = x^2 - 2x$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$$\boxed{x = 4} \quad \cancel{x = -2}$$

$$11.) 5 \ln(x) = 35$$

$$\ln x = 7$$

$$e^7 = x$$

$$12.) \ln(-3x-1) - \ln 7 = 2$$

$$\ln\left(\frac{-3x-1}{7}\right) = 2$$

$$e^2 = \frac{-3x-1}{7}$$

$$\frac{7e^2 + 1}{-3} = x$$

$$13.) 8^x = 20$$

$$\log_8 20 = x$$

$$14.) \log(5x-2)^2 = 4$$

$$2 \log(5x-2) = 4$$

$$\log 5x-2 = 2$$

$$10^2 = 5x-2$$

$$102 = 5x-2$$

$$102 = 5x$$

$$x = \frac{102}{5}$$

$$15.) e^{-x} = 5$$

$$\frac{-x}{-1} = \frac{\ln 5}{-1}$$

$$x = -\ln 5$$

$$16.) 7^{3x} = 18$$

$$\log_7 18 = 3x$$

$$x = \frac{\log_7 18}{3}$$

$$17.) 10^{4x-3} = 24$$

$$\log_{10} 24 = 4x-3$$

$$\frac{(\log_{10} 24) + 3}{4} = \frac{4x}{4}$$

$$x = \frac{\log 24 + 3}{4}$$

$$18.) \log(x) + \log(x-1) = \log 72$$

$$\log x(x-1) = \log 72$$

$$x^2 - x = 72$$

$$x^2 - x - 72 = 0$$

$$(x-9)(x+8) = 0$$

$$x = 9 \quad x = -8$$

Word Problem. An investment of \$1,750 earns 5.75% interest, which is compounded monthly. After approximately how many years will the investment be worth \$5,000?

$$P = 1750$$

$$r = .0575$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad n = 12$$

$$5000 = 1750\left(1 + \frac{.0575}{12}\right)^{12t}$$

$$2.857 = (1.00479)^{12t}$$

$$\log_{1.00479} 2.857 = 12t$$

$$219.61779 = 12t$$

$$t = 18.3 \text{ years}$$

A stock bought at an initial price of \$5 per share is worth \$12 after 7 years. Find the rate of continuously compounded interest in this investment.

$$P = 5$$

$$t = 7$$

$$12 = 5e^{r \cdot 7}$$

$$\ln \frac{12}{5} = \ln e^{7r}$$

$$A = Pert$$

$$\frac{\ln \frac{12}{5}}{7} = \frac{7r}{7}$$

$$\text{Rate} = 12.5\%$$

$$r = \frac{\ln \frac{12}{5}}{7} \quad r = .125$$