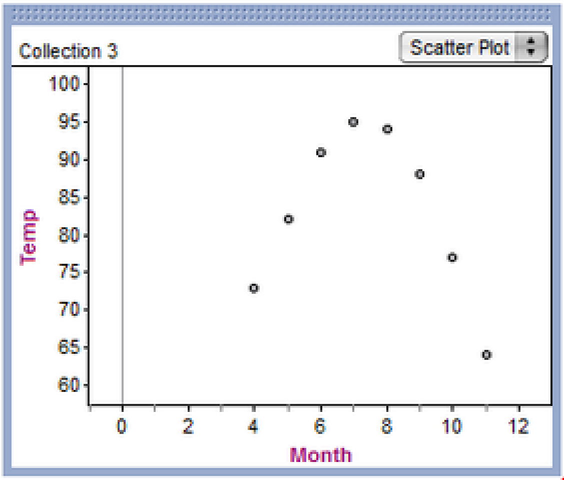
4 – Circular Functions

Problem Set 4-1

Problems 1 and 2 are a quick introduction to the chapter. Let's do these in class now and you can complete the rest of the problem set for homework.

1. Recall this data set from problem 2-7-1:
   The table and graph below give the average monthly temperatures (F) for Phoenix, AZ.



Collection 3

	Month	Temp
1	4	73
2	5	82
3	6	91
4	7	95
5	8	94
6	9	88
7	10	77
8	11	64

Source: *Phoenix Average Monthly Temperatures*. About.com, n.d. Web. 25 Feb. 2013. <http://phoenix.about.com/od/weather/a/averagetemps.htm>.

Answer all parts of the question in the context of the Phoenix weather data.

a) What is the interpretation of the ordered pair (7, 95)?
In the seventh month of the year, meaning July, the average temperature in Phoenix was 95º.

b) If the data set were to be extended, what would month 19 represent?
If we considered month 1 to be January of some year, then month 13 would be January of the following year and month 19 would be July of the following year.

c) What would you expect the average monthly temperature to be in month 19?
Assuming the average monthly temperatures from year to year are fairly similar, we would expect the average monthly temperature in month 19 to also be around 95º.

d) Describe the general pattern of the data over several years. What type of model would be appropriate for this data?
We would expect average monthly temperature to rise and fall over the course of each year, repeating approximately the same pattern each year.

e) What property of this data over the years makes it different than any data set we could model in Chapter 2?
The fact that the end behavior of the data is a repeating pattern, not approaching a certain value or trending to positive or negative infinity, makes it different than the data sets we could model in Chapter 2.

2. Name three phenomena that have values that increase and decrease repeatedly over time.
   Answers may vary.

3. Go to www.google.com/trends and find a word (not previously discussed) that has been Googled periodically and find the period. Then find a word that has *not* been Googled in any periodic pattern.
   Answers may vary.

4. Convert 1 degree into radians. Convert 1 radian into degrees.
   $\frac{1^0}{1}\cdot\frac{2\pi}{360^o}=\frac{2\pi}{360}$; $a\approx 0.017$ radians, so 1^o is approximately 0.02 radians
   $\frac{1\text{ rad}}{1}\cdot\frac{360^o}{2\pi\text{ rad}}=\frac{360^o}{2\pi}$; $b\approx 57.3^0$, so 1 radian is approximately 57^o

5. Convert 3.14 radians into degrees.
   $\frac{3.14\text{ rad}}{1}\cdot\frac{360^o}{2\pi\text{ rad}}=\frac{3.14\cdot 360^o}{2\pi}$; $d\approx 179.9^o$, so 3.14 radians, which is almost π,
   is approximately 179.9^o, which is almost 180^o as we would expect

6. Answer the following questions about radians:
   a) What is a radian? Is it a length? Explain.
   A radian is an angle measure that gives the ratio: $\frac{\text{length of the arc}}{\text{length of the radius}}$. **It is not a length; it is a ratio of two lengths.**

   b) Why are radians called radians?
   Radians are named thusly because they are a system of measurement that describes how many radii would fit into the arc intercepted by a given angle.

7. Fill in the missing angle measures in both degrees and radians. An example is given. Don't forget the quadrantal angles!

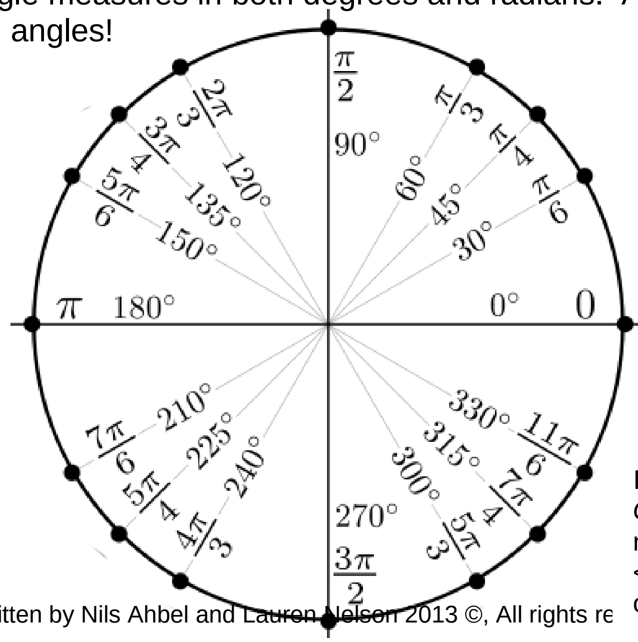


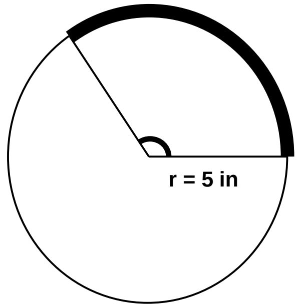
Image Source: *The Python Game Book*. Horst Jens, etc., n.d. Web. 25 Feb. 2013. <http://thepythongamebook.com/en:pygame:step017>.

8. Convert -270º into revolutions. $\frac{3}{4}$ of a revolution clockwise

9. Convert 1 counter-clockwise revolution into radians. 2π

10. The unit circle is defined as a circle centered at the origin with a radius of 1. What is true about the radian measure of any angle in a unit circle?
The radian measure of an angle in a unit circle is <u>equal</u> to the length of the arc that it subtends (intercepts).

11. Consider the circle below.

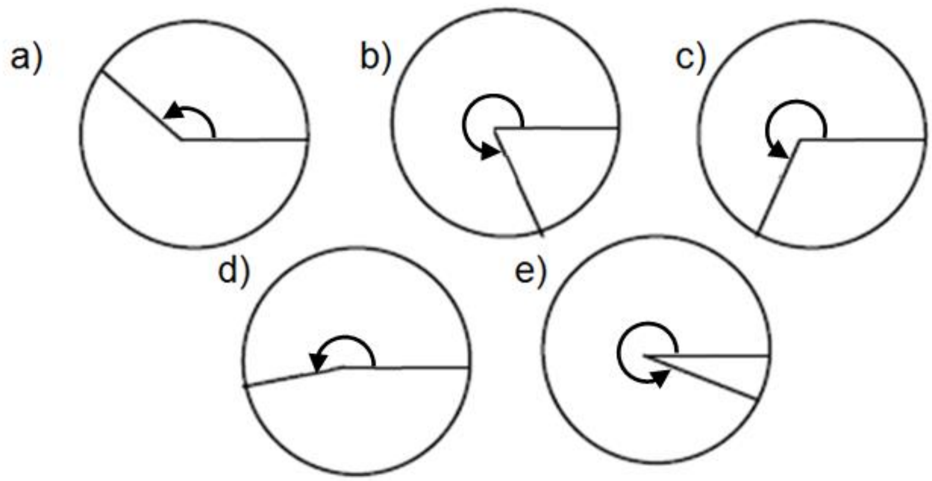


a) Using the radius as a scale, estimate the length of the arc in bold.
The arc marked in bold is about 10 units long.
(Answers may vary).

b) Using your estimate from part a, give the measure of the marked angle in radians.
The angle in radians is
$\frac{\text{length of the arc}}{\text{length of the radius}} \approx \frac{\text{10 inches}}{\text{5 inches}}$, so it is approximately 2 radians.

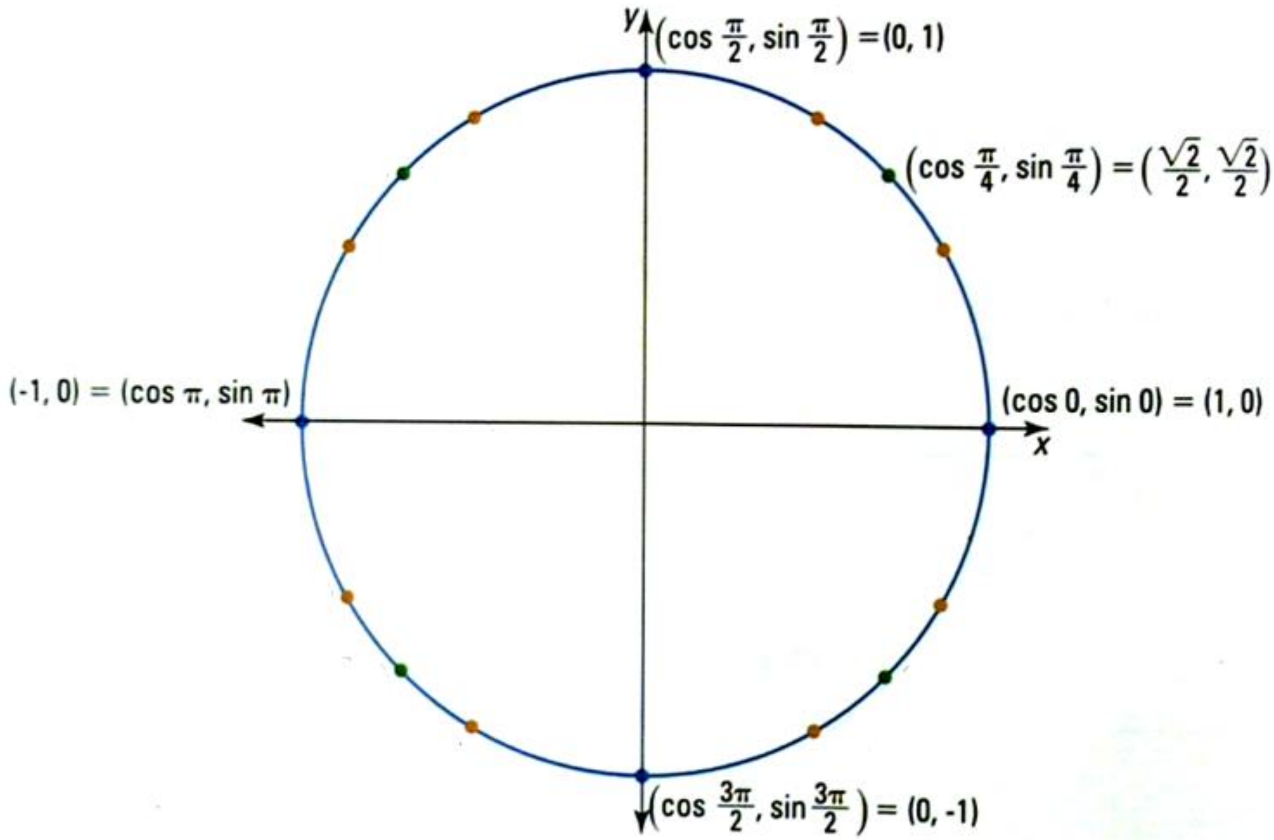
12. Note that angles measured in degrees are written as numbers followed by a degree symbol, but angles measured in radians have no symbol after the number (for example: $\frac{\pi}{2}$), indicating that radians have no units. Why is it that radians have no units or symbol?
A radian measure is a length divided by a length, so as we can see in 11 above, inches in the numerator cancels out with inches in the denominator leaving radians as a measure without units.

13. Estimate the radian measure of each angle shown:
a) 2.5 b) 5 c) 4.25 d) 3.4 e) 5.5
(Remember, those are estimates – answers may vary.)


a)
b)
c)
d)
e)

Problem Set 4-2

1. The figure below shows coordinates of the point (1, 0) after a rotation of $\frac{\pi}{2}$ radians about the origin. The coordinates are given both in terms of cos and sin as well as numbers. Fill in the missing coordinates in the same way for the point (1, 0) after a rotation of 0, $\frac{\pi}{4}$, π, and $\frac{3\pi}{2}$.



2. Refer to your answers in problem 1 to help you evaluate the following expressions for cos and sin. Use those values and the definition of tangent to evaluate the expressions for tangent.

a) $\cos(0) = \mathbf{1}$ $\sin(0) = \mathbf{0}$ $\tan(0) = \mathbf{0}$

b) $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\tan\left(\frac{\pi}{4}\right) = \mathbf{1}$

c) $\cos\left(\frac{\pi}{2}\right) = \mathbf{0}$ $\sin\left(\frac{\pi}{2}\right) = \mathbf{1}$ $\tan\left(\frac{\pi}{2}\right)$ **undefined**

d) $\cos(\pi) = \mathbf{-1}$ $\sin(\pi) = \mathbf{0}$ $\tan(\pi) = \mathbf{0}$

e) $\cos\left(\frac{3\pi}{2}\right) = \mathbf{0}$ $\sin\left(\frac{3\pi}{2}\right) = \mathbf{-1}$ $\tan\left(\frac{3\pi}{2}\right)$ **undefined**

3. Refer to your answers in problem 1 to help you answer the following questions:

a) Why is sin(0) = sin(π) ?
sin(θ) is defined as the y coordinate of the image of the point (1,0) after a rotation about the origin by θ. At 0 radians, the y coordinate is 0. At π radians, the y coordinate is also 0.

b) Which of the angles for which you found coordinates have the same cosine value and why?
The cosine is the same at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ because the x coordinates at both of those points are zero.

c) What range of angles between 0 and 2π have a positive sine value and why?
When the point (1, 0) is rotated between 0 and π (not including 0 or π) it would have a positive y coordinate, therefore the sine value is positive between 0 and π (not including 0 or π).

d) What range of angles between 0 and 2π have a negative cosine value and why?
When the point (1,0) is rotated between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ (not including $\frac{\pi}{2}$ or $\frac{3\pi}{2}$) it would have a negative x coordinate, therefore the cosine value is negative between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ (not including $\frac{\pi}{2}$ or $\frac{3\pi}{2}$).

e) Using the values of $\cos(0)$ and $\sin(0)$ and the definition of the tangent function, calculate $\tan(0)$. Do the same for $\tan\left(\frac{\pi}{2}\right)$, $\tan(\pi)$ and $\tan\left(\frac{3\pi}{2}\right)$.

$$\tan(0)=\frac{\sin(0)}{\cos(0)}=\frac{0}{1}=0 \qquad \tan\left(\frac{\pi}{2}\right)=\frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)}=\frac{1}{0}, \text{which is undefined}$$

$$\tan(\pi)=\frac{\sin(\pi)}{\cos(\pi)}=\frac{0}{-1}=0 \qquad \tan\left(\frac{3\pi}{2}\right)=\frac{\sin\left(\frac{3\pi}{2}\right)}{\cos\left(\frac{3\pi}{2}\right)}=\frac{-1}{0}, \text{which is undefined}$$

f) The coordinates at the angle $\frac{3\pi}{4}$ are not labeled in the diagram in (1), but use the coordinates at $\frac{\pi}{4}$ to make a conjecture about the coordinates of the point at $\frac{3\pi}{4}$.
When the point (1, 0) is rotated through an angle of $\frac{3\pi}{4}$, its image is a reflection of the point at $\frac{\pi}{4}$. $\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$ and $\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$. When reflected across the y axis the x coordinate becomes negative, so the coordinates of the point at $\frac{3\pi}{4}$ would be $\left(\cos\left(\frac{3\pi}{4}\right),\sin\left(\frac{3\pi}{4}\right)\right)=\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

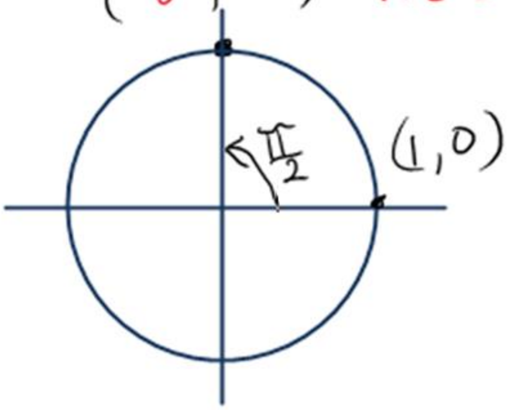
4. How can you express the exact coordinates of the image of (1, 0) after a rotation of 87º about the origin? **(cos(87º), sin(87º))**

5. Estimate the coordinates of the image of (1, 0) after a rotation of 87º about the origin by drawing a picture of the unit circle. Check to see how close your estimate is with a calculator by evaluating your answer in problem 4. $\approx$ (0.05, 0.999).

6. Draw a picture using the unit circle to explain why $\cos\left(\frac{\pi}{2}\right) = 0$.

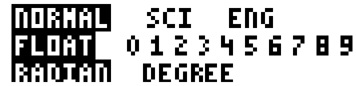
$\left(\cos\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$

$(0, 1)$ $\therefore \cos\frac{\pi}{2} = 0$



Problem Set 4-3

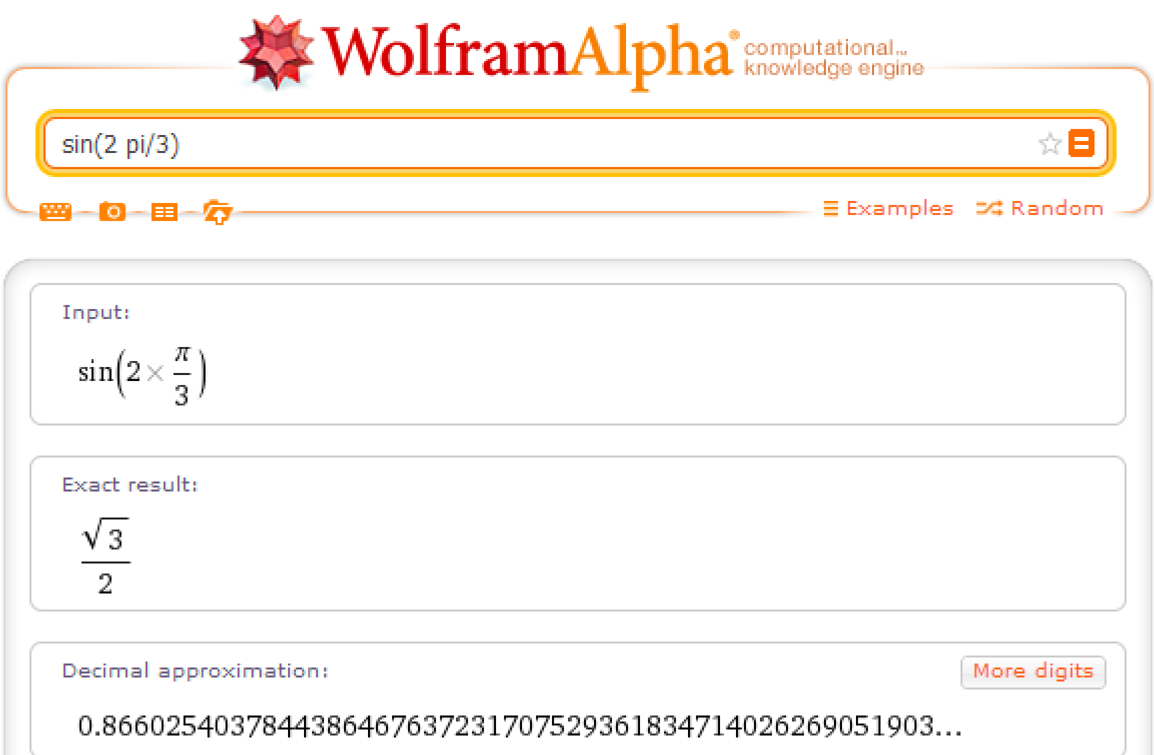
You can evaluate trigonometric expressions of special angles, but if the answer is irrational, a graphics calculator without CAS (Computer Algebra System) will give only an approximation. For example, make sure you are in "radian mode"



and evaluate $\sin\left(\frac{2\pi}{3}\right)$. The result is

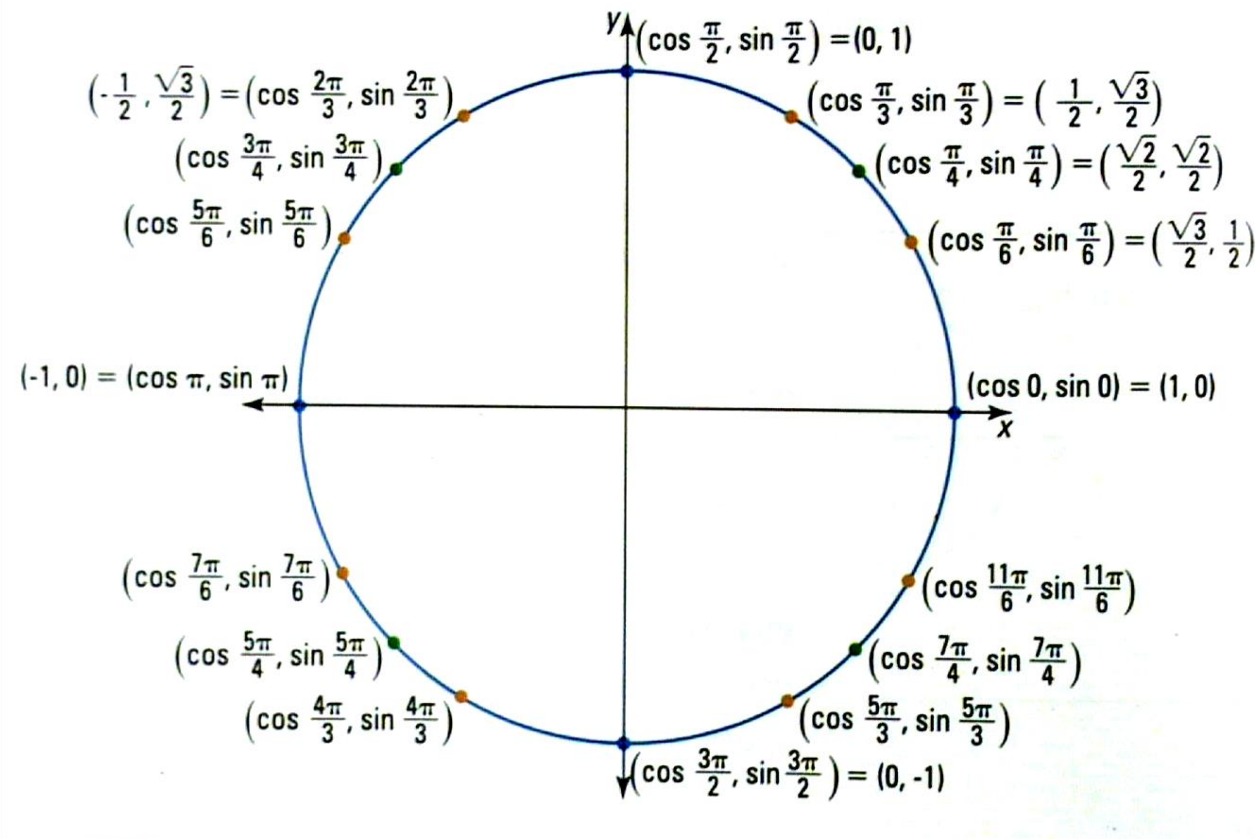
Note, the value given is an *approximation* of $\sin\left(\frac{2\pi}{3}\right)$ and not an *exact value*.

Wolfram Alpha will give you both the approximation and exact value as shown below. Because Computer Algebra Systems (like Wolfram Alpha) are available for free on the web, this unit has a decreased emphasis on knowing, for example, that $\sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}$ and an increased emphasis on explaining *why* $\sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}$.



WolframAlpha. Wolfram Research, n.d. Web. 26 Apr. 2013.
http://www.wolframalpha.com/input/?i=sin%282+pi%2F3%29.

1. The figure below has coordinates filled in for the point (1, 0) after a rotation of $\frac{2\pi}{3}$ radians about the origin. The coordinates are given both in terms of cos and sin as well as numbers. Fill in the missing coordinates in the same way for all "special angles". This may look like an onerous task, but there is a pattern to the values you're looking for which makes the task much easier. Make sure you understand the pattern instead of treating each coordinate as a separate problem.
Some coordinates in the 2nd - 4th quadrants are left for you to do.



2. Refer to your answer to problem 1 to help you answer the following questions:
   a) What are the possible values for the sine of any special angle?
   $0, \pm\frac{1}{2}, \pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{3}}{2}, \pm 1$
   b) What are the possible values for the cosine of any special angle?
   $0, \pm\frac{1}{2}, \pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{3}}{2}, \pm 1$
   c) Are the sine and cosine of a given angle ever the same between 0 and 2π? If so, where and why?
   The sine and cosine would be the same at an angle where the x and y coordinates are exactly the same. That would happen at $\frac{\pi}{4}$ and $\frac{3\pi}{4}$, the angles that lie along the line y = x.

d) Why is $\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right)$?

When the point (1,0) is rotated through an angle of $\frac{5\pi}{6}$, its image is a reflection over the y-axis of the point at $\frac{\pi}{6}$, so it will have the same y coordinate.

$\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$**, so** $\sin\left(\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{3}$**.**

e) Using the values of $\cos\left(\frac{\pi}{4}\right)$ and $\sin\left(\frac{\pi}{4}\right)$ and the definition of tangent, calculate $\tan\left(\frac{\pi}{4}\right)$. Do the same for $\tan\left(\frac{3\pi}{4}\right)$, $\tan\left(\frac{5\pi}{4}\right)$ and $\tan\left(\frac{7\pi}{4}\right)$.

$$\tan\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1 \qquad \tan\left(\frac{3\pi}{4}\right) = \frac{\sin\left(\frac{3\pi}{4}\right)}{\cos\left(\frac{3\pi}{4}\right)} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = -1$$

$$\tan\left(\frac{5\pi}{4}\right) = \frac{\sin\left(\frac{5\pi}{4}\right)}{\cos\left(\frac{5\pi}{4}\right)} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = 1 \qquad \tan\left(\frac{7\pi}{4}\right) = \frac{\sin\left(\frac{7\pi}{4}\right)}{\cos\left(\frac{7\pi}{4}\right)} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = -1$$

3. Refer to your answer to problem 1 to help you evaluate the following expressions:

a) $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

c) $\sin\left(\frac{11\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

d) $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

e) $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$ $\quad$ $\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\quad$ $\tan\left(-\frac{2\pi}{3}\right) = \sqrt{3}$

f) $\cos(180^\circ) = -1$ $\quad$ $\sin(180^\circ) = 0$ $\quad$ $\tan(180^\circ) = 0$

g) $\cos(-135^\circ) = -\frac{\sqrt{2}}{2}$ $\quad$ $\sin(-135^\circ) = -\frac{\sqrt{2}}{2}$ $\quad$ $\tan(-135^\circ) = 1$

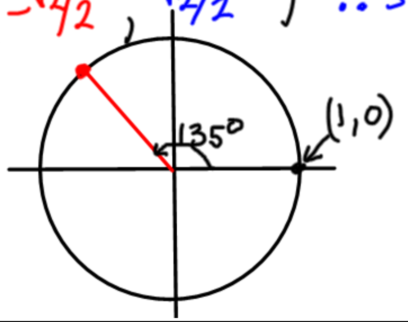
h) $\cos(300^\circ) = \frac{1}{2}$ $\quad$ $\sin(300^\circ) = -\frac{\sqrt{3}}{2}$ $\quad$ $\tan(300^\circ) = -\sqrt{3}$

4. Draw a picture that includes the unit circle to explain why…

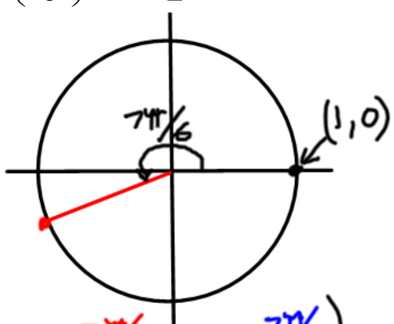
a) $\sin(135^0) = \dfrac{\sqrt{2}}{2}$

$(\cos 135^\circ, \sin 135^\circ)$

$\left(-\sqrt{2}/2, \sqrt{2}/2\right)$ $\therefore \sin 135^\circ = \dfrac{\sqrt{2}}{2}$



b) $\cos\left(\dfrac{7\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$



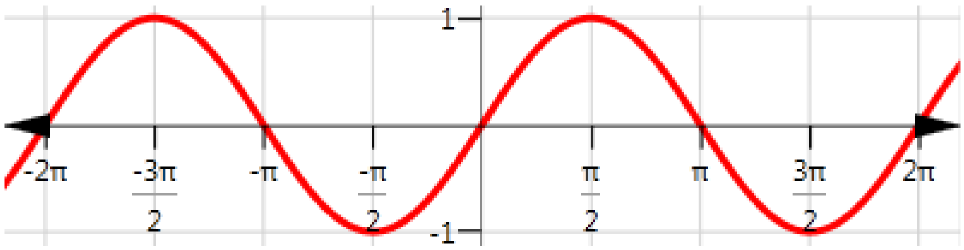
$(\cos 7\pi/6, \sin 7\pi/6)$

$\left(-\sqrt{3}/2, -1/2\right)$ $\therefore \cos \dfrac{7\pi}{6} = -\dfrac{\sqrt{3}}{2}$

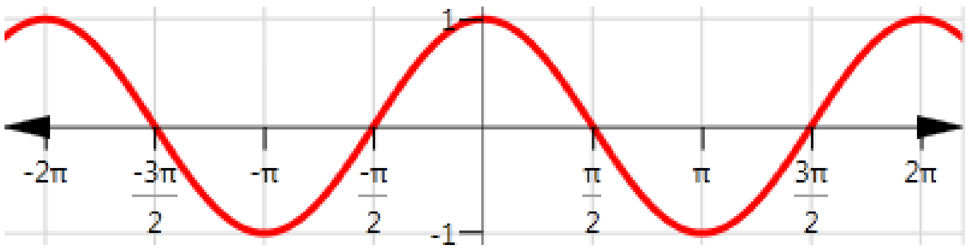
Problem Set 4-4

You need to be able to do 1 and 2 neatly, accurately, and without a calculator. Practice so that you can draw each graph in about 1 minute.

1. Sketch $y = \sin(x)$ for $-2\pi \le x \le 2\pi$.

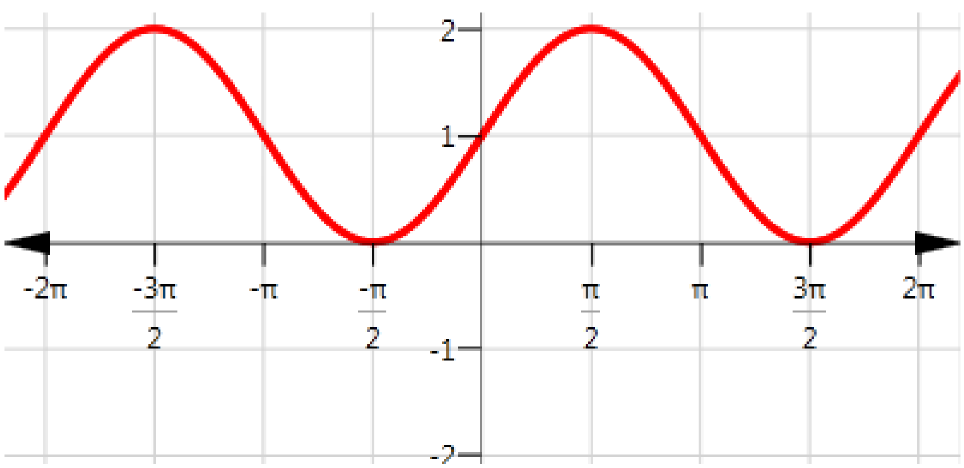


2. Sketch $y = \cos(x)$ for $-2\pi \le x \le 2\pi$.

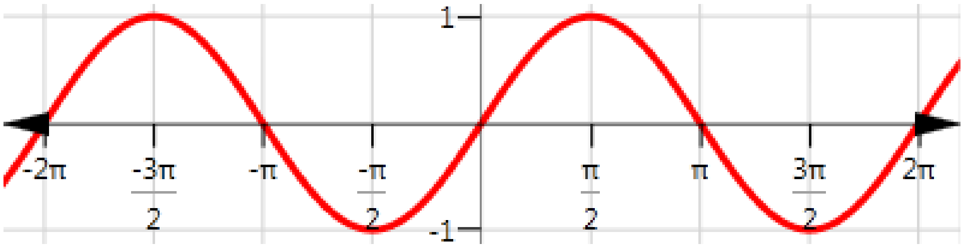


Recall from Chapter 3 that we graphed $y = x^2$ and $y = (x-3)^2$ and determined what the effect was of replacing x with x-3. Similarly, we graphed $y = x^2$ and $y - 1 = x^2$ and determined what the effect was of replacing y with y-1. We called the generalization of that principle the Graph Translation Theorem. Use that principle to sketch the graphs in (3) and (4). Sketch the graphs without your calculator and then confirm you have the correct answer with your calculator.

3. Sketch $y - 1 = \sin(x)$, which is the same as $y = \sin(x) + 1$, for $-2\pi \le x \le 2\pi$.

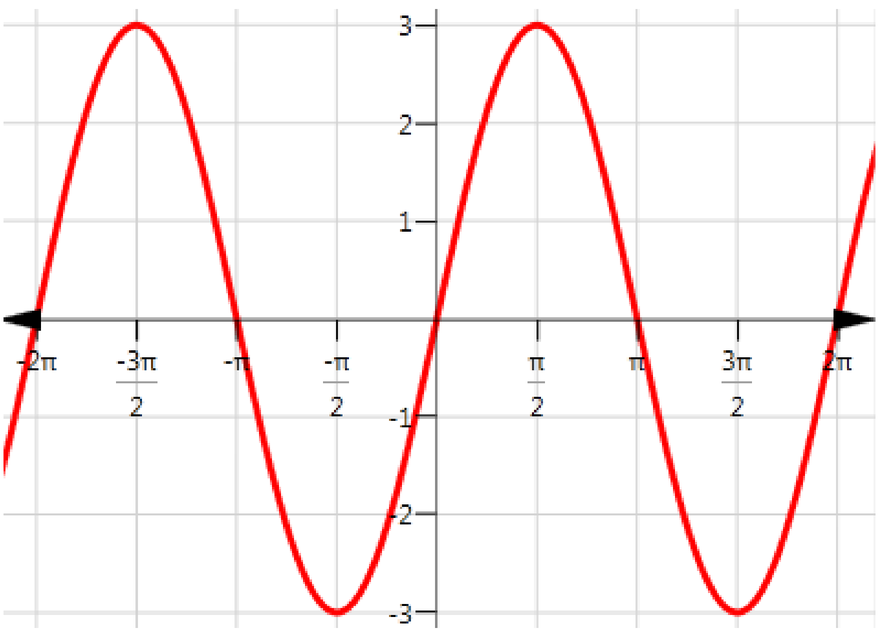


4. Sketch $y = \cos\left(x - \frac{\pi}{2}\right)$, for $-2\pi \le x \le 2\pi$.

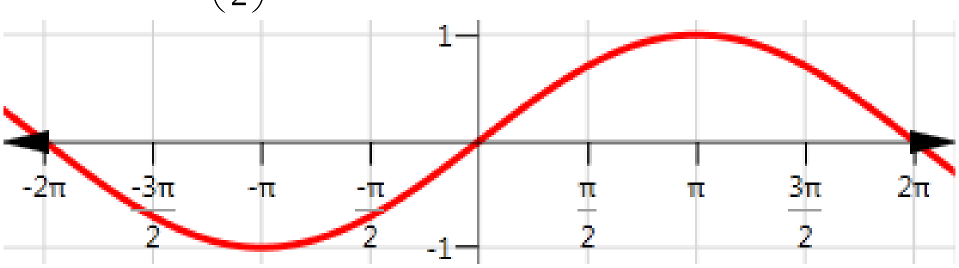


Also, recall from Chapter 3 that we graphed $y = x^2$ and $\frac{y}{3} = x^2$ and determined what the effect was of replacing y with $\frac{y}{3}$. We called the generalization of that principle the Graph Scale Change Theorem. Use that principle to sketch the graphs in 5 and 6.

5. Sketch $\frac{y}{3} = \sin(x)$, which is the same as $y = 3\sin(x)$, for $-2\pi \le x \le 2\pi$.



6. Sketch $y = \sin\left(\frac{x}{2}\right)$, for $-2\pi \le x \le 2\pi$.



7. For the function $f(x)=\sin(x)$:
   a) What is the domain? **All Real Numbers**
   b) What is the range? **$-1 \le y \le 1$**
   c) For what values of *x* in the range $-2\pi \le x \le 2\pi$ is the function increasing?
   $-2\pi \le x < -\frac{3\pi}{2}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $\frac{3\pi}{2} < x \le 2\pi$

8. For the function $f(x)=\cos(x)$:
   a) What is the domain? **All Real Numbers**
   b) What is the range? **$-1 \le y \le 1$**
   c) For what values of x in the range $-2\pi \le x \le 2\pi$ is the function decreasing?
   $-2\pi < x < -\pi$, $0 < x < \pi$

9. What is the largest output value (value of y) of y = sin(x) – 3? Explain how you got your answer.
   The largest output of $y=\sin(x)$ is 1. The graph of $y=\sin(x)-3$ is translated down 3 units as compared to the graph of $y=\sin(x)$, so the largest output of $y=\sin(x)-3$ is -2.

10. What is the largest output value (value of y) of y = 4cos(x)? Explain how you got your answer.
    The largest output of $y=\cos(x)$ is 1. The graph of $y=4\cos(x)$ is stretched vertically by a factor of 4 as compared to the graph of $y=\cos(x)$, so the largest output of $y=4\cos(x)$ is 4.

Problem Set 4-5

1. What is the definition of $\tan(x)$?

   $\tan(\theta)=\dfrac{\sin(\theta)}{\cos(\theta)}$, where $\cos(\theta)\neq 0$

2. Why is $\tan\left(\dfrac{\pi}{4}\right)$ = 1?

   Because $\sin\left(\dfrac{\pi}{4}\right)=\cos\left(\dfrac{\pi}{4}\right)$, so $\dfrac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}=1$

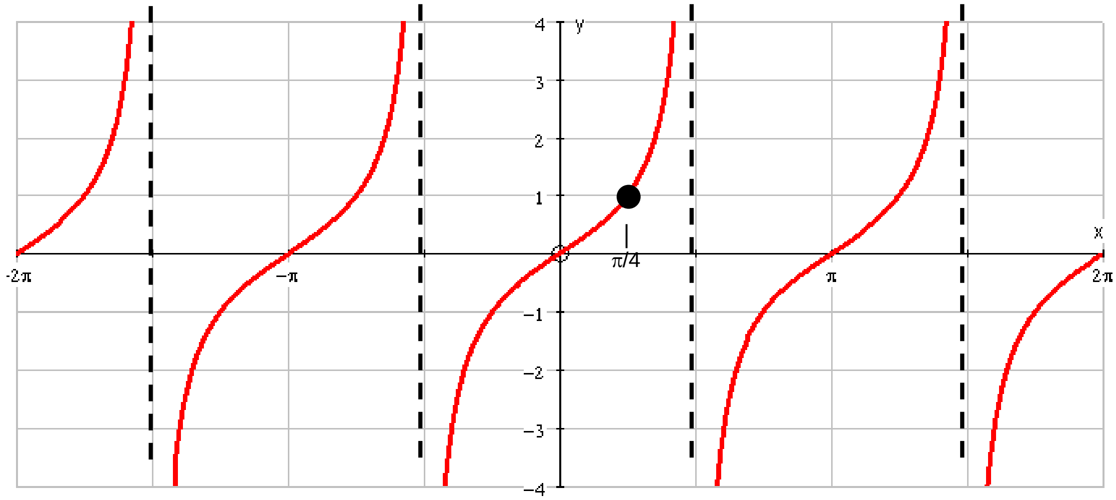
3. The following is a review question from Problem Set 4-3.
   Evaluate the following expressions:

a) $\cos(0)=$ **1**	$\sin(0)=$ **0**	$\tan(0)=$ **0**
b) $\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$	$\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$	$\tan\left(\frac{\pi}{4}\right)=$ **1**
c) $\cos\left(\frac{\pi}{2}\right)=$ **0**	$\sin\left(\frac{\pi}{2}\right)=$ **1**	$\tan\left(\frac{\pi}{2}\right)$ **undefined**
d) $\cos\left(\frac{3\pi}{4}\right)=-\frac{\sqrt{2}}{2}$	$\sin\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{2}$	$\tan\left(\frac{3\pi}{4}\right)=$ **-1**
e) $\cos(\pi)=$ **-1**	$\sin(\pi)=$ **0**	$\tan(\pi)=$ **0**
f) $\cos\left(\frac{5\pi}{4}\right)=-\frac{\sqrt{2}}{2}$	$\sin\left(\frac{5\pi}{4}\right)=-\frac{\sqrt{2}}{2}$	$\tan\left(\frac{5\pi}{4}\right)=$ **1**
g) $\cos\left(\frac{3\pi}{2}\right)=$ **0**	$\sin\left(\frac{3\pi}{2}\right)=$ **-1**	$\tan\left(\frac{3\pi}{2}\right)$ **undefined**
h) $\cos\left(\frac{7\pi}{4}\right)=\frac{\sqrt{2}}{2}$	$\sin\left(\frac{7\pi}{4}\right)=-\frac{\sqrt{2}}{2}$	$\tan\left(\frac{7\pi}{4}\right)=$ **-1**
i) $\cos(2\pi)=$ **1**	$\sin(2\pi)=$ **0**	$\tan(2\pi)=$ **0**

4. For what values of x on the interval $-2\pi \le x \le 2\pi$ is $\tan(x)$ undefined? Explain why.

   $-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ **Those are angles where the cosine value is 0, making the tangent ratio undefined because its denominator is 0.**

5. Sketch $y = \tan(x)$ for $-2\pi \le x \le 2\pi$.



6. Looking at your graph from problem 5, you can see that the tangent function is periodic.
   a) What is the period of the tangent function?
      Is it the same as the period of sine and cosine?
      Recall that the mathematical definition of period refers to functions where $f(x+P) = f(x)$. Sine and cosine are periodic with $P = 2\pi$. While the statement $\tan(x+2\pi) = \tan(x)$ is true, the tangent function actually repeats itself more often. We can see from the graph that the tangent function repeats itself every π radians, therefore the fundamental/prime period of tangent is π.
   b) Show that $\tan\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4} + \pi\right)$.

      Note that $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$.

      $$\tan\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1 \qquad \tan\left(\frac{5\pi}{4}\right) = \frac{\sin\left(\frac{5\pi}{4}\right)}{\cos\left(\frac{5\pi}{4}\right)} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = 1$$

7. Find the exact value of $\tan(100.5\pi)$. **Undefined (Note: you should be able to explain how you got your answer to this question without your calculator)**

8. For the function $f(x) = \tan(x)$
   a) What is the domain? **{All Real Numbers x: x ≠ $\frac{\pi}{2}$ + πn} where n is any integer**
   b) What is the range? **All Real Numbers**

Problem Set 4-6

This problem set will probably take two days.

1. Learning the theory – Either in class or for homework, view two different resources that will explain how to model with the function $y = \sin(x)$. Note: the theory is explained using y = sin(x) as the modeling function, but y = cos(x) could just as easily be used; it is a matter of personal preference.
   a) Modeling with y=sin(x) (FluidMath video)
   b) Modeling with y=sin(x) (PPT)

2. Practice the theory – There are four resources that will help you practice the theory. The first two are PowerPoint files
   a) Modeling with y=sin(x) and y=cos(x) - **exact** model practice (PPT)
   b) Modeling with y=sin(x) - **approximate** model practice (PPT)
   Note: This second PowerPoint uses y = sin(x) as the modeling function, but y = cos(x) could just as easily be used.

   The last two are Fathom files
   c) Modeling with y=**sin**(x) - approximate model practice (Fathom)
   d) Modeling with y=**cos**(x) - approximate model practice (Fathom)

In 3 – 5, complete parts a – e and then any additional parts of that problem.
Note: you can choose to use either $y = \sin(x)$ or $y = \cos(x)$ as the parent function.

a) Find an equation in implicit form $\frac{y-k}{a} = \sin\left(\frac{x-h}{b}\right)$ that models the data.

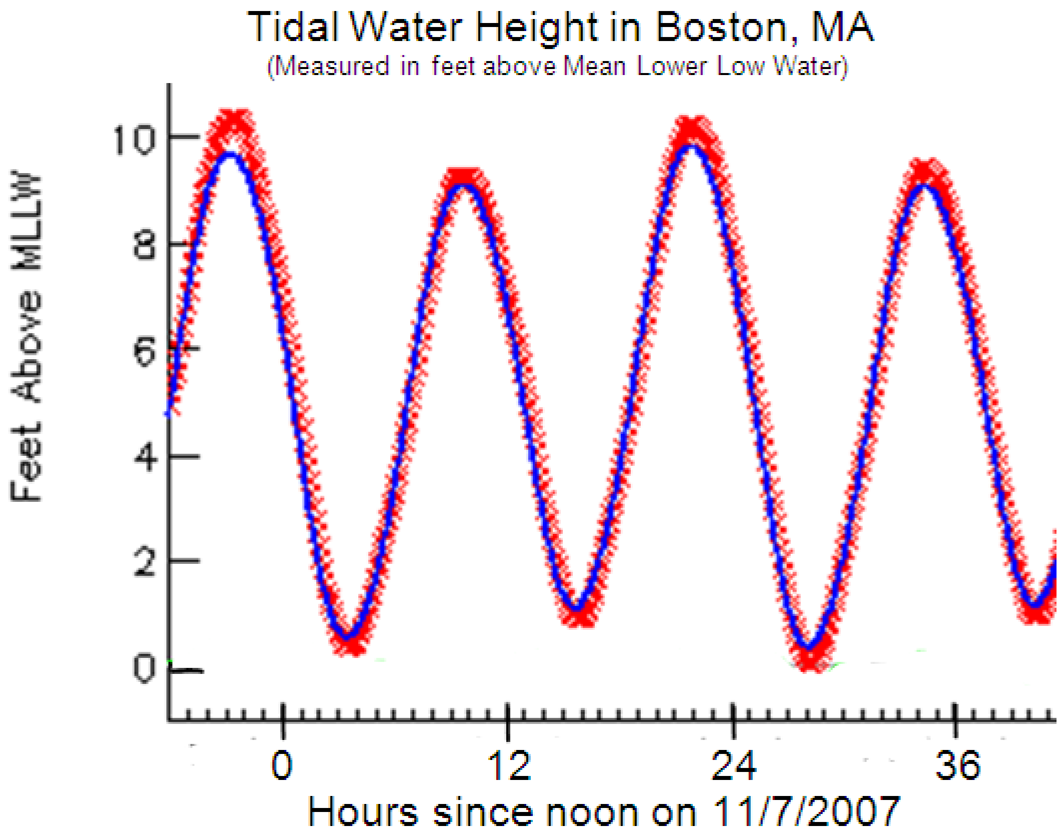
b) Rewrite the equation in function form $y = a \cdot \sin\left(\frac{x-h}{b}\right) + k$.

c) Enter your equation from part c into your calculator and graph it on the same window as the graph shown in the problem. They should look approximately the same.

d) Give the period and amplitude for your model and explain what they mean within the context of the problem.

e) Give the coordinates of the point (h, k) for your model and explain what they mean within the context of the problem.

3. For the tidal water graph below, do parts a – e as outlined above and then answer two more questions:
   f) Use the graph to estimate how many feet above Mean Lower Low Water (MLLW) the tide was 24 hours after noon on 11/7/2007.
   g) Use your equation from part b to <u>calculate</u> how many feet above MLLW the tide was 24 hours after noon on 11/7/2007.



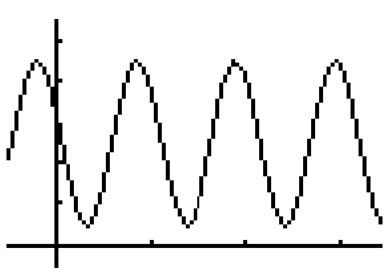
Source: *Water Level Stations by State*. NOAA, n.d. Web. 7 Nov. 2007. <http://tidesonline.nos.noaa.gov/ geographic.html>.

a) $\frac{y-5}{4} = \sin\left(\frac{x-7}{2}\right)$ **(aswers may vary slightly)**

b) $y = 4 \cdot \sin\left(\frac{x-7}{2}\right) + 5$ **on the TI-84 it looks like** $Y_1 = 4*\sin\left(\frac{X-7}{2}\right)+5$

c)

```
WINDOW
 Xmin=-6
 Xmax=41
 Xscl=12
 Ymin=-1
 Ymax=11
 Yscl=2
↓Xres=1
```



d) **Period – The period is about 12.5 which means that according to the model there are about 12.5 hours between one low tide and the next or between one high tide and the next.**

Amplitude – The amplitude is about 4 which means that according to the model there is about 8 ft (2 x 4) difference between the height of ocean at high tide and low tide in Boston, MA.

e) **Point – The model goes through the point (7, 5) which means that according to the model, 7 hours after noon on 11/7/2007 the height of the water was 5 ft above MLLW.**

f) Use the graph to estimate how many feet above MLLW the tide was 24 hours since noon on 11/7/2007. **About 8 feet.**

g) Use your equation from part b to estimate how many feet above MLLW the tide will be 24 hours since noon on 11/7/2007.

$4 \cdot \sin\left(\frac{24-7}{2}\right) + 5 \approx 8.2$ **About 8.2 feet**

4. For the average daily low temperature in Concord, NC graph below (the blue curve), do parts a – e as outlined above and then answer three more questions:

   f) Martin Luther King, Jr. Day is January 20th, the 20th day of the year. Use the graph to estimate the mean low temperature in Concord, NC on MLK Day.

   g) Use your equation from part b to calculate the mean low temperature in Concord, NC on

   h) Recall that a function is periodic with period P if $f(x+P)=f(x)$ for all values of x. Show that your model is periodic by demonstrating that $f(x+P)=f(x)$ for January 15th (x=15). Do this with the equation, not the graph.

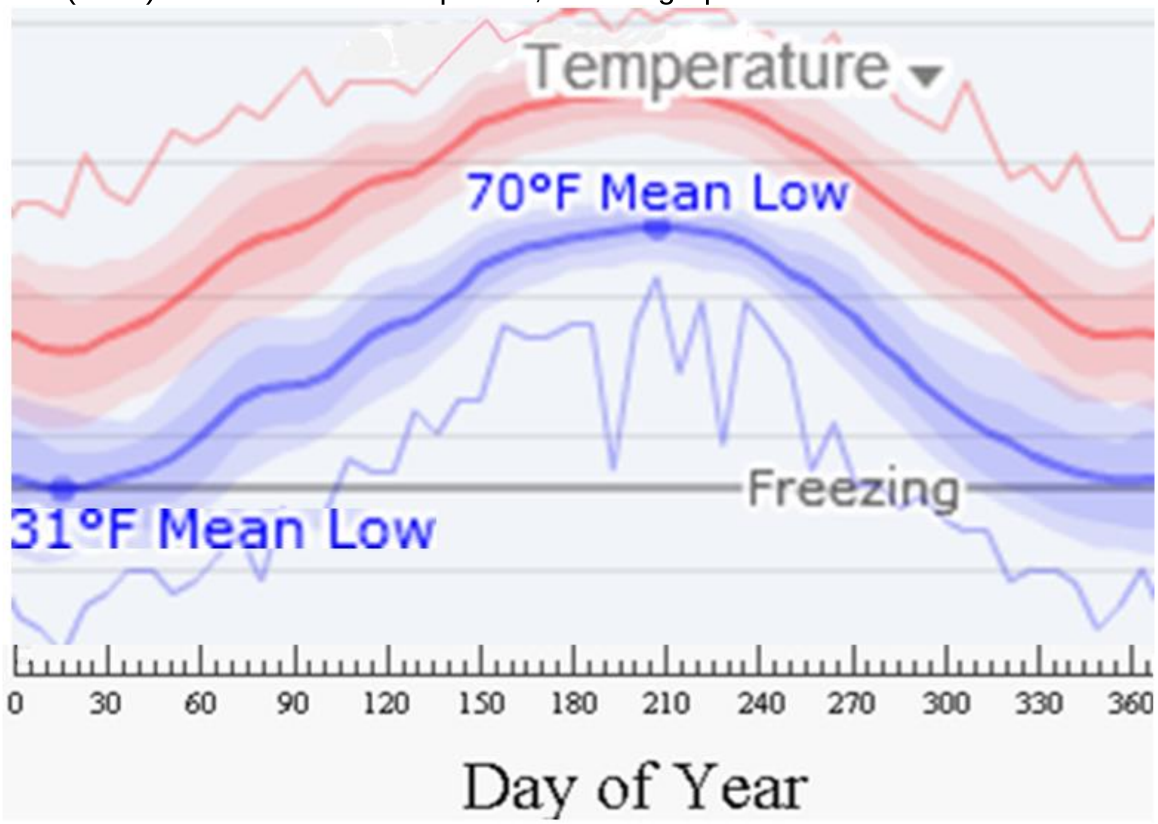
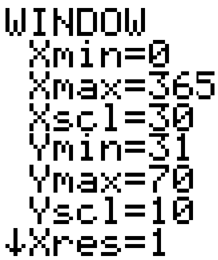


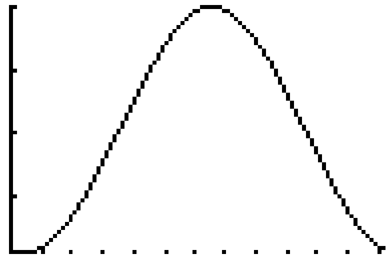
Image source: *Concord, NC Weather*. WeatherSpark, n.d. Web. 21 Feb. 2013 <http://weatherspark.com/#!dashboard;a=USA/NC/Concord>.

a) $\frac{y-50.5}{19.5}=\sin\left(\frac{x-105}{58.1}\right)$ **(answers may vary slightly)**

b) $y=19.5\cdot\sin\left(\frac{x-105}{58.1}\right)+50.5$

c)





d) **Period – The period is about 365, which means that according to the model there are about 365 days in the cycle of temperature swings.**

Amplitude – The amplitude is 19.5 which means that according to the model there is about 39 ^{0}F (2 x 19.5) difference between the maximum mean low temperature and the minimum mean low temperature.

e) **Point – The model goes through the point (105, 50.5) which means that according to the trig model on 105^{th} day of the year, the mean low temperature in Concord, NC is about 50.5 ^{0}F.**

f) Martin Luther King, Jr. Day is January 20^{th}, the 20^{th} day of the year. Use the graph to estimate the mean low temperature in Concord, NC on MLK Day.
About 31 ^{0}F.

g) Use your equation from part b to calculate the mean low temperature in Concord, NC on MLK Day. **31.11 ^{0}F (input x = 20).**

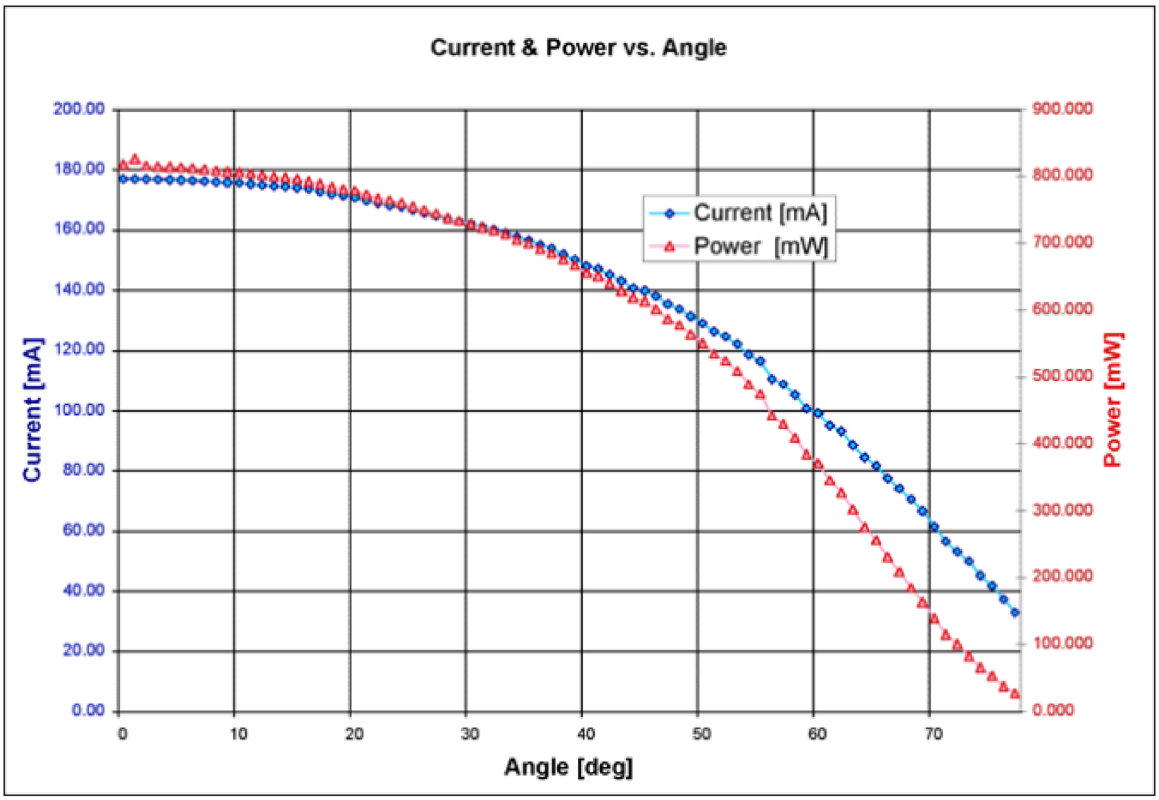
h) Recall that a function is periodic with period P if $f(x+P)=f(x)$ for all values of x. Show that your model is periodic by demonstrating that $f(x+P)=f(x)$ for January 15^{th} (x=15). Do this with the equation, not the graph.
This is left for you to do.

5. A solar cell will generate its maximum current when the cell is aligned to 0 degrees (perpendicular to the Sun's rays) and the cell will generate no current when the cell is aligned to 90 degrees (parallel to the Sun's rays). The **blue** scatterplot below gives the generated current in mA for various alignments in degrees. Do parts a – e as outlined above and then answer three more questions:

   f) According to your model, what is the maximum current this solar cell will generate?

   g) How much current will the cell generate at 45 degrees?

   h) What percentage of the maximum current will the cell generate at 45 degrees?



Source: *Solar Panel*. Rob Roberts, Camilo Jimenez, Naji Ghosseiri, n.d. Web. 23 Feb. 2013. <http://userwww.sfsu.edu/ozer/engr300-solar1N.pdf>.

a) $\frac{y-0}{178} = \sin\left(\frac{x+90}{57.3}\right)$ (answers may vary)

b) $y = 178 \cdot \sin\left(\frac{x+90}{57.3}\right)$

c) **Part c is left for you to do on your own**

d) **Period – The period is about 360. It is challenging to give meaning to the period in this case. We can say that one quarter of the period is 90 degrees and so according to the model as the sun's rays move 90 degrees, from perpendicular to parallel to the solar cell, the current goes from a maximum to zero.**

Amplitude – The amplitude of the trig model is 178 which means that according to the trig model there is about 178 mA difference from the maximum current to the minimum current, which is zero.

e) **Point – The model goes through the point (-90, 0) which means that according to the trig model when the alignment is -90 degrees (sun's rays parallel to the cell, the current generated is 0.**

f) According to your model, what is the maximum current this solar cell will generate? **178 mA**

g) How much current will the cell generate at 45 degrees? **125.9 mA**

h) What percentage of the maximum current will the cell generate at 45 degrees? **About 71% (hmmm, 71% or 0.71 looks familiar)**

6. In 3-5, can you conclude that you have found an <u>appropriate</u> model?
While the models may appear to fit the data well and we can be confident that periodic models are suitable, we can't conclude that the models are appropriate until we look at residuals.

Problem Set 4-7

Complete steps a-d for each of the data sets below using Fathom.

a) If possible, find a sinusoidal modeling equation for the data and graph your function over the data plot in Fathom.

b) Give the period and amplitude of your model and explain what they mean within the context of the data.

c) Give the coordinates of the point (h, k) for your model and explain what they mean within the context of the data.

d) Make a residual plot and comment on the appropriateness of your model.

The following data sets are posted in
Course Material → 4-7: Modeling with Circular Functions

1. 💻 Average Monthly Temperature for Worcester, MA
Independent variable – Month number
Dependent variable – Average monthly temperature

$$y = 23.3 \cdot \sin\left(\frac{x - 4.02}{2.025}\right) + 45.8$$

2. 💻 Deerfield Sunrise and Sunset Times
Independent variable - Month number
Dependent variable - Daylight_minutes

$$y = 184.4 \cdot \sin\left(\frac{x - 3.63}{1.925}\right) + 731$$

3. 💻 Pizza Prices at UC Davis
Independent variable – Pizza diameter
Dependent variable - Price
While a useful sinusoidal model can be found for this data, when we consider the expected end behavior we realize that a sinusoidal model could never be appropriate for this data. Pizza price would not be periodic; it should continue to rise in proportion with the area of the pizza (in^2), meaning a quadratic model would be more appropriate.

4. 💻 Refrigerator Temperatures
Independent variable - Time
Dependent variable – Temperature

$$y = 1.03 \cdot \sin\left(\frac{x - 15.85}{2.72}\right) + 36.1$$

5. Satellite Orbit

Independent variable – Orbit time

Dependent variable - Distance from the Earth

$$y = 4615 \cdot \sin\left(\frac{x - 0}{19.1}\right) + 0 \quad \text{or} \quad y = 4615 \cdot \sin\left(\frac{x}{19.1}\right)$$

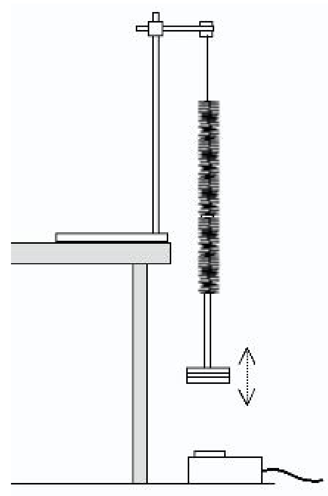
In addition to parts a-d described above, answer one additional question.

e) According to your model (use the equation), how far north or south of the equator is the satellite 45 minutes into its orbit?

About 3,263 miles north

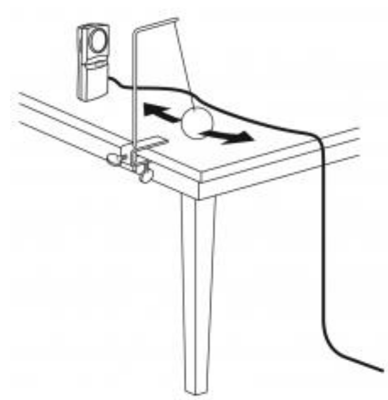
Problem Set 4-8

Data for 1 and 2 will be collected in class. Complete the same parts a-e that you did in problem set 4-6 for each situation, using Fathom to "analyze the data". Be sure to include a residual plot.



1. Weight on a spring
   Independent variable – Time
   Dependent variable – Height (vertical distance from the sensor)

   Image source: *Investigating a Mass on a Spring*. Nuffield Foundation, n.d. Web. 22 Feb. 2013. <http://www.nuffieldfoundation.org/sites/default/files/images/ Investigating%20a%20mass-on-spring%20oscillator_322.jpg>.



2. Pendulum
   Independent variable – Time
   Dependent variable – Horizontal distance from the sensor

   Image source: *Pendulum Motion*. Vernier, n.d. Web. 22 Feb. 2013. <http://www.vernier.com/images/cache/ lab.RWV-22-DQ-tic_toc.424.238.png>.

3. CO_2 data 1980-1990
   Independent variable – Year
   Dependent variable – CO_2 concentration
   Fit a model to the data. This will require you to "compose" two functions together. It is challenging, but you can do it.

Problem Set 4-9

All of the problems in this set come from the following source:
The North Carolina School of Science and Mathematics. *Contemporary Precalculus Through Applications*. 2nd Edition, 1999 ed. N.p.: Everyday Learning Corp., n.d. Print.

1. Suppose a Ferris wheel has radius 33.2 feet and makes three complete revolutions every minute. For clearance, the bottom of the Ferris wheel is 4 feet above the ground.
   a) Sketch a graph that shows how a particular passenger's height above the ground varies over time as he or she rides the Ferris wheel. Assume that the passenger is at the bottom of the Ferris wheel when $t = 0$.
   b) Write a function that models the passenger's height above the ground as a function of time.

2. In Los Angeles on the first day of summer (June 21) there are 14 hours and 26 minutes of daylight, and on the first day of winter (December 21) there are 9 hours and 54 minutes of daylight. On average there are 12 hours and 10 minutes of daylight; this average amount occurs on March 20 and September 22.
   a) Sketch a graph that displays this information. Use number of months after June 21 as the independent variable and minutes of daylight as the dependent variable.
   b) Write an equation that expresses d, the amount of daylight per day in minutes, as a function of t, the number of months after June 2.

3. A population of lynx oscillates in a four-year cycle. Kate is a biologist who keeps records of the lynx population in a small area. She has counted lynx on January 1st of each year. Her first count in 1994 was 40; in 1995, 60; in 1996, back to 40; and in 1997, 20. The population was back to 40 in 1998.
   a) Sketch a graph that shows the lynx population as a function of time. Assume that the relationship is sinusoidal.
   b) Write a function that expresses the lynx population as a function of time.

4. In a tidal river, the time between high tide and low tide is approximately 6.2 hours. The average depth of the water at a port on the river is 4 meters; at high tide the depth is 5 meters.
   a) Sketch a graph of the depth of the water at the port over time if the relationship between time and depth is sinusoidal and there is a high tide at noon.
   b) Write an equation for your curve. Let t represent the number of hours after noon.